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THE SUMARIO COMPENDIOSO
Sumario cópeditoso delas quetas
de plata y oro en los reynos del Perú son necessarias a los mercaderes y todo genero de tratantes. Ló algunas reglas tocantes al Arithmetica.

Fecho por Juan Díez Freyle.
THE SUMARIO COMPENDIOSO
OF BROTHER JUAN DIEZ

THE EARLIEST MATHEMATICAL WORK
OF THE NEW WORLD

BY

DAVID EUGENE SMITH

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PREFACE

If the student of the history of education were asked to name the earliest work on mathematics published by an American press, he might, after a little investigation, mention the anonymous arithmetic that was printed in Boston in the year 1729. It is now known that this was the work of that Isaac Greenwood who held for some years the chair of mathematics in what was then Harvard College. If he should search the records still farther back, he might come upon the American reprint of Hodder’s well-known English arithmetic, the first textbook on the subject, so far as known, to appear in our language on this side the Atlantic. If he should look to the early Puritans in New England for books of a mathematical nature, or to the Dutch settlers in New Amsterdam, he would look in vain; for, so far as known, all the colonists in what is now the United States were content to depend upon European textbooks to supply the needs of the relatively few schools that they maintained in the seventeenth century.

The earliest mathematical work to appear in the New World, however, antedated Hodder and Greenwood by more than a century and a half. It was published long before the Puritans had any idea of migrating to another continent, and fifty years before Henry Hudson discovered the river that bears his name. Of this work there remain perhaps only four copies, and it is desirable, not alone because of its rarity but because of its importance in the history of education on our continent, that a record of the text should be made generally accessible.

In making the translation the original methods of expression have been followed in many cases in which smoother diction would have suggested greater freedom of rendition. The reason has been, in general, the desire to bring such expressions as "8 per cent," "8 per 100," "one cosa and $\frac{1}{2}$ a cosa," and "four and $\frac{1}{2}$" into sharp contrast with the corresponding ones of the present time, and to show the great advance in symbolism and in methods of solution and proof. Such minor errors as were common in a century of careless proofreading have usually been corrected without comment. Aside from this, a certain freedom of translation has been assumed for the evident purpose of aiding the reader to follow the spirit of the text.

The editor wishes to express his indebtedness to Señorita Carolina Marcial Dorado for her scholarly assistance in the translation of the Spanish text.

DAVID EUGENE SMITH
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INTRODUCTION
THE MEXICO OF THE PERIOD

In order to understand the *Sumario Compendioso* it is necessary to consider briefly the political and social situation in Mexico in the middle of the sixteenth century. Cortés entered the ancient city of Tenochtitlan, later known as Mexico, in the year 1519, but its capture and destruction occurred two years later, in 1521. Thus, in the very year that Luther was attacking certain ancient privileges in the Old World, the representatives of other ancient privileges were attacking and destroying a worthy civilization in the newly discovered continent.

The rebuilding of the city began at once, and the new capital soon entered upon an era of great prosperity, disturbed, however, by the failure of Cortés to show the power as a civil leader that he had shown in his military capacity. The first viceroy of New Spain, which included the present Mexico, was a man of remarkable genius and of prophetic vision,—Don Antonio de Mendoza. He assumed his office in 1535, and for fifteen years administered the affairs of the colony with such success as to win for himself the name of "the good viceroy." He founded schools, established a mint, ameliorated the condition of the natives, and encouraged the development of the arts. In his efforts at improving the condition of the people he was ably assisted by Juan de Zumárraga, the first Bishop of Mexico. Among the various activities of these leaders was the arrangement made with the printing establishment of Juan Cromberger of Seville whereby a branch should be set up in the capital of New Spain.

Mendoza became viceroy of Peru in 1550 and died in Lima in 1552. Upon leaving Mexico he was succeeded by Don Luís de Velasco, a member of an illustrious Castilian family and one who labored faithfully for the betterment of the people intrusted to his charge. One of the first steps taken by him was to found, in 1551, the Real y Pontificia Universidad de la Ciudad de Mexico, and he was at all times interested in the success of the press and in the work of its manager, Juan Pablos, as the name appears in the books of the period. Don Luís died in 1564, sincerely mourned by the people, and was laid to rest in the Monastery of Santo Domingo, in the city in which he had exercised his benevolent authority.

In the same year that Mendoza left Mexico for Peru, Zumárraga passed away, his death being a genuine loss to the State as well as the Church. In the following year Alonso de Montufar* was nominated as his successor and was

*The spelling is substantially as given in the facsimile on page 62. The first name usually appears as Alphonso or Alfonso.
consecrated in 1553. For sixteen years he presided with great success over the Church in New Spain, and five years after the death of Don Luis de Velasco he too was buried within the precincts consecrated to the memory of Santo Domingo. It was in his time and with his sanction that the *Sumario Compendioso* was issued from the press, and well did he deserve the praise accorded to him by a contemporary writer in the words


Tempora mutantur, nos et mutamus in illis. Such words of praise, expressed in the most sonorous of tongues and in words that seem exaggerated to our ears, belong to the past. In our rapid, unsettled, materialistic life we seem to take pride in our neglect of dignity of eulogy, excusing ourselves by condemning the past as insincere in its praise. Who that reads the story of this early period, however, can say that such descriptions of the characters and accomplishments of those who carried the Cross to the New World were exaggerated, or that they failed to express the genuine sentiments of the people to whose spiritual needs these brothers of the holy orders so conscientiously ministered?
II

PRINTING ESTABLISHED IN MEXICO

The idea of setting up a press in Mexico seems to have been considered as early as 1534, even before Mendoza became viceroy, doubtless at the suggestion of Juan de Zumárraga; but it was not until 1536 that the plan was carried out. Juan Cromberger then sent over as his representative Juan Pablos, a Lombard printer, and so the "casa de Juan Cromberger" was established, prepared to spread the doctrines of the Church to the salvation of the souls of the unbelievers. Cromberger himself never went to Mexico, but his name appears either on the portadas or in the colophons of all the early books. From and after 1545, however, the name is no longer seen, Cromberger having died in 1540.

It was this John Paul who printed the Sumario Compendioso, in 1556, and in order that the significance of the work may be the better appreciated it is appropriate to mention the following books, known to have been printed by him before that year:

- 1537. Escala Espiritual para llegar al cielo, possibly in 1537, but there is no copy extant.
- 1539. Breve y más compendiosa doctrina Christiana en lengua Mexicana y Castellana.
- 1540. Manual de Adullos.
- 1541. Relacion d'I espátable terremoto.
- 1543. Doctrina breve.
- 1544. Tripartito del Christiantssimo y consolatorio doctor Juan Gerson. This contains the earliest woodcut printed in Mexico.
- c. 1544. A second edition of the preceding work.
- 1544. Doctrina xpiana.
- 1545-1546. Doctrina cristiana.
- 1546. Doctrina xpiana.
- 1546. Doctrina cristiana.
- 1546. Cancionero Spiritual. The first book to bear the name of Juan Pablos as the printer, — "Juá pablos Lóbardo."
- 1547. Regla christiana breue.
- c. 1547. Doctrina cristiana en lengua mexicana.
- 1548. Doctrina Cristiano.
- 1548. Ordenãças y copilacion de leyes.
- 1548. Doctrina Cristiano en Lengua Huasteca.
- 1550. Doctrina cristiana.
- 1553. Doctrina cristiana.
- 1554. Recognitio, Summularum.
- 1554. Dialecta resolutio cum textu Aristotelis.
- 1554. Dídlogos, by Cervantes (Francisco Salazar).
- 1555. Vn vocabulario en la lengua Castellana y Mexicana.
In 1556 five books were published, among them the *Sumario Compendioso*. It thus appears that not only was this the first book on mathematics, but it was the first textbook of any kind, except for religious instruction, to be published outside of Europe.

In his *Bibliografia Mexicana del Siglo XVI*, Icazbalceta speaks of a copy in the library of the Convento de la Merced and of one in the Ramírez sale. There is also one in the Biblioteca Nacional at Madrid, from which three folios are missing, and it is this copy that has been used in the preparation of the present work, the missing portion containing parts of tables not included in this edition. There is also a copy in the British Museum.

The author of the *Sumario* was Juan Diez, a native of the Spanish province of Galicia, a companion of Cortés in the conquest of New Spain, and the editor of the works of Juan de Avila, known as “the apostle of Andalusia,” and of the *Itinerario* of the Spanish fleet to Yucatan in 1518. He is sometimes confused with Juan Diaz, a contemporary theologian and author. In a letter written to Charles V in 1533 he is mentioned as a “clérigo anciano y honrado,” so that he must have been advanced in years when the *Sumario* appeared. That this was the case is also apparent from a record of the expedition of 1518 in which it is stated that “triximus vn clerigo que dezia joan diaz,” doubtless a young and adventurous apostle, full of zeal and desire to make known the gospel in the New World.

The other four books appearing from this press in the year 1556 are as follows:

*Costituciones del Arzobispado y Provincia de la muy insigne y muy leal ciudad de Tenuxtitlan México de la Nueva España;*

*Costituciones Fratrum Heremitarum Sancti patris nostri Augustini Hiponensis Episcopi et doctoris Ecclesiae;*

*Speculum Conjugiorum;*

*Catecismo y Doctrina Cristiana en Idioma Ullateco.*

Not again in the sixteenth century did the Mexican printers publish any work on mathematics, except for a brief *Instrucción Nautica* which appeared in 1587. The press was generally true to its early purpose to issue only books relating to the conversion of the native inhabitants to the way of the Cross.
III

GENERAL DESCRIPTION OF THE BOOK

The *Sumario Compendioso* consists of one hundred and three folios, generally numbered. After the dedication (folios i, v, and ij, r) there is an elaborate set of tables, including those relating to the purchase price of various grades of silver (folio iij, v), to per cents (folio xlix, r), to the purchase price of gold (folio lvij, v), to assays (folio [lxxxj, r]), and to monetary affairs of various kinds.

The mathematical text (folio xcj, v) consists of twenty-four pages besides the colophon (folio ciij, v). Of these pages, eighteen relate chiefly to arithmetic and six to algebra.

The signatures are a (j, ..., iij [ ..., viij]), and similarly for b, ..., i, k, l, m, n (j, ..., iij [ ..., viij]).

Folio i, r consists of the arms as shown in the facsimile, and the title: *Sumario cōpēdioso delas quētas / de plata y oro q en los reynos del Piru son necessarias a los mercaderes: y todo genero de tratantes. Cō algunas reglas tocantes al Arithmetica. Fecho por Juan Diez freyle.*

Folios i, v and ij, r contain the dedication to the viceroy, beginning:

*Al Illustrissimo Señor Don Luys / de Velasco Visorrey y gouernador d'la nueva España. &c. Juan diez freyle: que perpetua felicidad le dessea.*

As to himself the author says:

"Por quéto Juã diez freyle estāte pre / sente enesta ciudad de Mexico me a becho relaciō q l cō ci ã cu y / dado trabajo & industria a cōpuesto vn libro de quētas de plata & / oro cō algumas reglas t' uera del ordinario: tocātes al arismetica: el q l es de / mucha utilidad & puecho pa en los reynos del Piru a causa d'las muchas / variedades q enel ay enlas leyes de plata & oro & otras cosas q alla le vsã / lasqles todas estan en el dicho libro muy copiosamēte puestas."

He therefore undertook the work for the purpose of assisting those who were engaged in the buying of the gold and silver which was already being taken from the mines of Peru and Mexico for the further enriching of the moneyed class and the rulers of Spain. The author felt that he could best serve this purpose by preparing such a set of tables as should relieve these merchants as far as possible from any necessity for computation. For this he had very good precedent, not so much in Spain as in Italy. In 1503 Anton Bartholomeo di Paxi had published in Venice a *Tariffa de pexi e mesvre* containing numerous tables relating to weight, value, and the like, and intended for the Venetian merchants engaged in foreign trade; in 1535 Giovanni Mariani had published a *Tariffa perpetua* in the same city and intended for a similar purpose among the merchants of all of Northern
Italy; and besides these, various other works of a similar nature had already been issued with the intention of relieving merchants from the extensive calculations imposed upon them by the complex systems of measures then in use.

Apparently prompted by the further demand for a brief treatment of arithmetic which should be suited to the needs of apprentices in the counting houses of the New World, the author devotes eighteen pages to the subject of computation and presents it in a manner not unworthy of the European writers of the period.

The most interesting feature of the work, however, is neither the tables nor the arithmetic; it consists of six pages devoted to algebra, chiefly relating to the quadratic equation.

The reason for this interest will be appreciated the more when we consider the state of algebra in Europe in the middle of the sixteenth century. Puzzle problems involving numbers, such as would now be solved by algebra, were known to the Egyptians in the second millennium B.C.; but no treatise upon the theory of equations is known before about A.D. 275, when Diophantus wrote his great work. It is not until the beginning of the ninth century that the word algebra appears in its present sense, having first been used by al-Khowárizmī in a treatise written in Bagdad in the time of the caliphs.

In the Middle Ages there appeared a number of algebraists of ability, notably Leonardo Fibonacci of Pisa, who lived early in the thirteenth century; and little by little these scholars added to the store of material which had already accumulated in the works of the later Greeks and the Orientals.

With the advent of printing from movable types, in the second half of the fifteenth century, there was awakened a new interest in mathematics, and particularly in the field of algebra. The Greek and oriental writers had solved the quadratic equation, but the equations of the third and fourth degrees still awaited solution, and a better symbolism was in urgent demand.

The middle of the sixteenth century saw the solution of the cubic and biquadratic equations by the Italian algebraists, and saw numerous efforts made at devising a convenient symbolism.

It was at this time that Juan Diez wrote. There had already appeared the notable algebra of Cardan (the Ars Magna of 1545), the Germans had published two treatises of merit, and there had appeared in 1514, from the pen of Gillis Vander Hoecke, a Dutch mathematician, a work of some consequence; but the number of treatises printed before 1556 was small, and these were far from being popular. It is therefore of considerable interest to know that an obscure writer in Mexico should have produced even six pages on the subject at this early period in the development of printed scientific literature.

8
IV

NATURE OF THE TABLES

The general nature of the tables may be seen from the facsimile on page 10. The abbreviations used are as follows:

 prá is used for peso and pesos, originally a certain weight of metal, like pound, libra, and lira. The word comes from the Latin pensum, from pendere, "to hang." From the same root we have such words as poise, which also appears in avoirdupois, and such physical terms as pendant and pendulum. The Castilian libra, which found its way into Mexico, was about 1.014 avoirdupois pounds.

 t is used for tomin and tomínes. The tomin was the eighth part of a peso, and this was the same as the later real. The peso was therefore a "piece of eight." The tomin was also \( \frac{1}{8} \) of a peso of weight, or \( \frac{1}{3} \) of a drachm. The name comes from the Arabic tomn, "an eighth part."

 m̃ros is used for maravedi and maravedís, a word derived from the name of the Moorish dynasty, Murābitin, during which the coin was first struck. In these tables 56 maravedí make 1 tomin, but in tables of a later period the real (tomín) is given as equivalent to 34 maravedís de plata Mexicanos. In some of the tables of Juan Diez the tomin is taken as 56\( \frac{1}{4} \) maravedis (fol. lvij), and there are several other slight variations of this kind in the tabular work. The maravedi is also used as a weight, as on page 10.

 on is used for onça, our ounce, from the Latin uncia, "a twelfth part," the ounce being the twelfth part of a Roman and early Spanish pound. From the same root we have our inch, the twelfth part of a foot. The uncial script of the Middle Ages received its name from the same source.

 g̃os is used for grano and granos, our grain as a unit of weight, one twelfth of a tomin; — "12. granos ñ tien vn tomin."

 U is often used for 1000. Thus, we have ijU for 2000, iijU for 3000, and so on. The name for U is cuento, given in the tables as cueto, a word derived from contar, "to reckon." This use of U was common in Spain in the sixteenth century and has an interesting history. The symbol may be seen in the last two lines of the facsimile on page 10, where 300 maravedís correspond to jU pesos, that is, to 1000 pesos. The U in this sense is of uncertain origin. It appears a century earlier as U and may possibly have come from one of the several Roman symbols for a thousand. Among the curious variants are D with the vertical bar duplicated, and a symbol resembling the late Greek character for 900. In the sixteenth century the Portuguese used for the same purpose a symbol, the cifrao, which somewhat resembled our present dollar sign.

9
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*Plutade. mil. d. deley so. iii*
The approximate nature of the tables may be seen from the page here shown in facsimile. In the third line the value of 1 ounce is given as 0 pesos 3 tomines 18\(\frac{3}{4}\) maravedis. The half ounce should then be worth half of this, or 1 tomin 37\(\frac{1}{2}\) maravedis, as stated. The quarter ounce should then be worth half this amount, or 46\(\frac{7}{8}\) maravedis, whereas it is given in the table as only 46 maravedis. Similar instances of a lack of exactness are found throughout the tables, — a fact that would hardly have been considered significant in the somewhat crude financial transactions of the period.

The Roman numerals are used in all the tables, as was the custom among many bankers in various parts of Europe until the close of the seventeenth century. Where the chief commercial and financial operations consisted in additions and subtractions, these numerals were nearly as convenient for purposes of practical computation as the Hindu-Arabic symbols in use to-day.

The tables extend to “dos mil. cccc. de ley.” There is a table of per cents extending to 30%. In this there are such entries as

\[
\text{iij. por. ciento. c. } \overset{\text{ps}}{\text{iij. }} \overset{\text{ps}}{\text{iij.}}
\]

that is, 3\% of 100 pesos is 3 pesos.

In general it may be said that the tables give the value of various numbers of ounces of silver in pesos, tomines, and maravedis.

The terms \textit{pesos}, \textit{tomines}, \textit{maravedis}, and \textit{varas} seem more acceptable in the translated text than any English words, and hence have been used. The more familiar marks, grains, ounces, crowns, and ducats have been given in English.

The tables are no longer of any importance, but as a matter of interest a single page is here shown in facsimile. Only the mathematical text has any historic significance, and it is this that appears in the translation.
THE TEXT
WITH TRANSLATION AND NOTES
Reglas ordinarias.

El que bastantemente testigo puesto por donde sin hazer quenta se pueda saber el valor de qual-
quier varra o tejo de plata o oro por diferente
ley et peso que tenga et el valor delos enteres
ques seacolumbran a dar por qualquier plata
0oro hasta treinta por ciento, y asi mismo el va-
lor de quallesquier pesos de plata corriente com-
prados de ensayado razonando el enteres de ocha a veinte por ci
ento juntamente con todo lo mas necesario delanueva España co
las reducciones de pesos vucados y coronas, de aqui adelante pon-
dre algunas reglas delas necesarias en los regnos del Peru juntaa-
mente con algunas quistiones para curiosos entre las cuales van
algunas delarte mayor reseruadas al algebra: las quales conlo de-
mas sino suere tal como conviene recebid la voluntad y sea carita-
tivamente emendado de la falta que tuviere.
Common Rules

NOW that I have sufficiently explained,* without doing the actual computing, how the value can be found of any ingot or bar of silver or gold of whatever standard or weight, and how to find the amount of commission up to thirty per cent which it is customary to give for any gold or silver; and, in the same way, how to ascertain the value of divers weights of silver currency bought as assayed, reckoning the commission from eight to twenty per cent, together with all else that is necessary in regard to the reduction of pesos, ducats, and crowns in New Spain,—from here on, I shall set forth some of the necessary rules which are used in the kingdom of Peru, together with certain problems for those who are interested, among which are certain parts of the arte mayor † pertaining to algebra. If these with the rest do not entirely meet with the approval of the reader, may he accept my good intentions, and, in as kindly a spirit as possible, excuse the mistakes which I may have made.

* In the tables, which make up the greater part of the book.
† The arte mayor was a term commonly used in the sixteenth century for algebra. It appears in the Latin of the period as ars magna and in the Italian as l'arte maggiore. Cardan, for example, called his great work on algebra by the name of Ars Magna, the work appearing at Nürnberg only eleven years before the Sumario Compendioso was published.

The name was occasionally combined with the ancient title, "The Science of Dark Things," used by Ahmes, an Egyptian mathematician of c. 1550 B.C., and with the Arabic title al-Jabr w'al Muqabalah, used by al-Khowârizmi, c. 820. An illustration of this is seen in the title of Gosselin's treatise, De Arte Magna, seu de occulta parte numerorum quae et Algebra et Almucabala vulgo dicitur, libri IV, which appeared in Paris in 1577.

The use of l'arte maggiore for higher arithmetic (algebra) as distinguished from l'arte minore for elementary arithmetic may have been suggested by the seven arti maggiori and the fourteen arti minori of the merchants of medieval Florence.

The Italians also called the science by the name Regola de la cosa, the reason being that the unknown quantity was called the cosa, as stated on page 51. Because of this fact the German algebraist Rudolf (1525) called his treatise Die Coss, and English writers of the same century spoke of algebra as the "cossike arte."

The Arabic title given above means "restoration and equation," and hence algebra came also to mean "restoration to health." It is for this reason that, in Don Quixote, they sent for "un algebrista who attended to the luckless Samson."
Reglas ordinarias.

Capítulo primero: Por el cual se da a entender la regla para hacer de plata corriente ensayada.

Atendido tengo que pocas veces será necesario hacer quenta que pase delos veinte por ciento que esta escripto: pero dezado esto a parte dare aquí la regla para que consaber partir la haga quien quiera: y es que ala cantidad q quieres saber quanto es de ensayado añadirás adelante dos zeros o cifras como estas, 00, y des pues añada con ciento el interés q das por lo ensayado, por lo qual parte aquello a q añadiste las dos cifras, y lo que saliere ala partición seran los pesos ensayados a q se bueule lo corriente. Y nota q lo que sobzare enla partición son pesos, y q los has de multiplicar por 2.8. tomínes que tiene vn peso y lo produzido has de partir por el partido de antes y el aduenimiento sera tomínes: y ansi mismo si algo sobzare son tomínes y has los de multiplicar por 12. granos q tiene vn tomín y partir los por el mismo partido; y el aduenimiento sera granos los quales pon con los pesos y tomínes de las particiones de antes y aquello sera lo que vale la plata corriente bulta en ensayada. Y sea te aviso que si en lo corriente quiere tomínes que poz cada vn tomín podrás en lugar delas dos cifras. 12. y medio porque mejorelo entiendas pondre aquí vn exemplo.

Primero Exemplo.

Digo que no tengo 4321. p. 8. 6. tomínes de plata corriente los quales quiero cópiar de plata ensayada o de ozo que melo da a 2.4 por ciento de ynteres para lo qual tengo dicho que ala cantidad corriente has de añadir dos cifras y si quiere tomínes poz cada uno en lugar delas cifras. 12. y medio poz q las cifras poz si no valé nada en lugar delas cifras. 12. y medio poz q las cifras poz si no valé nada
Common Rules

Chapter I, in which is explained the rule for finding the value of assayed silver.

I

UNDERSAND that sometimes it will be necessary to make calculations above the prescribed twenty per cent; but aside from this I shall now give a rule which anyone who knows division can follow, namely: to the amount of money with which you wish to buy the assayed silver annex two zeros or ciphers (00); then compute on a basis of a hundred the commission which you give for the assaying, and divide by this the number with the two ciphers annexed; the result of the division will be the assayed pesos which the currency will buy. It should be noticed that the remainder left from the division represents pesos and must be multiplied by 8, the number of tomines in a peso; this product must be divided by the same divisor as before, the quotient being the number of tomines. In the same way if there is again a remainder, it represents tomines and must be multiplied by 12, the number of grains in a tomin, and if we divide this by the same divisor, the quotient will be the number of grains. Now put these with the pesos and tomines and the result will be the value of silver currency in assayed form. Let me also say that if you wish the currency in tomines, put in place of the ciphers 12 and a half for each tomin, and in order that you may better understand I give an example.*

First example

Suppose that I have 4321 pesos 6 tomines in silver currency with which I wish to buy as much assayed silver or gold as they will give me at 24 per cent commission.† Now to the amount of currency you must annex two ciphers; and if you wish tomines, for each one put in place of the ciphers 12 and a half, because the ciphers themselves are not of any value when considered alone.

* The author here makes an approach to the decimal fraction. A tomin is \(\frac{1}{10}\) of a peso, and hence 4321 pesos 6 tomines is equal to 4321 pesos plus \(6 \times 0.12\frac{1}{2}\) pesos. It follows that 4321 pesos 6 tomines is equal to \((432,100 + 6 \times 12\frac{1}{2})\) hundredths of a peso. If, now, we wish to divide 4321 pesos 6 tomines by 1.24, we may avoid decimal fractions by dividing 432,175 by 124. This is what the author does in the illustrative problem which follows.

† The word *ynteres* (interest) is used by the author to mean any kind of percentage.
Reglas ordinarias.

Y puestas así delante sirven para aumentar en tal manera alas de atrás que al uno hacen valer ciento y así poniendo el valor de un to- 
mín o dos o cualesquier tomines sirven poz si y poz las dos cifras 
por cuanto tienen los grados que son unidad y vezna: y así 
medio aumenta al uno y se a 100. Y nota que esto no es otra cosa 
que multiplicar po2100. Y que se pone así po más brevedad: pues 
es tomando a nuestro ejemplo tan veys que lo corriente es. 4321, 
peños.6. tomines por los cuales pon adelante en lugar de las cifras 
75, cue su valor de los seis tomines a doce y medio cada uno y vie 
nen a ser. 432175, los cuales parte po2 ciento y veinte y quatro 
que son el valor de los 100. y el enterece y venírse a a la partición. 
3485. peños. y sobran.35. los cuales haz tomines quedam 
duplicando los pos.8. y son. 280. que partidos pos.124 vienen.2.to. 
y sobra.32. los cuales haz granos que es multiplicando los pos.12 
quien ve tomin y son.384. que partidos poz ciento y veinte y 
quattro te vendrán.3. granos que juntos con lo de mas son. 
3485.p8.2.to.3.granos.y esto es en lo que liquidamente se toman 
los. 4321, peños.6.tomines de corriente comprados de ensayado 
o de oro a 24. poz ciento y si quieres ver si es verdad añade les su 
interes allos.24. poz.100. que son.836. peños.3. tomines.9. gros 
y venírte a verílimo.

<table>
<thead>
<tr>
<th>Tomines de Corriente</th>
<th>Partición</th>
<th>Partidos</th>
<th>Granos de Ensayado</th>
<th>Enterece</th>
</tr>
</thead>
<tbody>
<tr>
<td>4321</td>
<td>121</td>
<td>206</td>
<td>06293</td>
<td>17055</td>
</tr>
<tr>
<td>32 sobra</td>
<td>1</td>
<td></td>
<td>432175/3485.2.to.3.9.</td>
<td></td>
</tr>
<tr>
<td>poz.12.9.</td>
<td>022</td>
<td></td>
<td>124444</td>
<td>35 sobra.3</td>
</tr>
<tr>
<td>son.384</td>
<td>384</td>
<td></td>
<td>1222</td>
<td>042 to.</td>
</tr>
<tr>
<td></td>
<td>124</td>
<td></td>
<td>11 son.280</td>
<td>280</td>
</tr>
</tbody>
</table>
Common Rules

but are annexed merely to raise the number in such a way that one becomes a hundred; and substituting the value of as many tomines as you wish, they serve, for themselves and for the two ciphers, to advance the number two places, through units and tens, thus raising one to 100. Observe also that this is nothing more than multiplying by 100 and is stated in this way for brevity. Now, using our example, we observe that the amount is 4321 pesos 6 tomines. In place of the two ciphers to be affixed, substitute 75, the value of six tomines, each being twelve and a half hundredths, and the result is 432,175. Divide this by one hundred and twenty-four, the value of the 100 plus the commission, and the result of the division is 3485 pesos, with a remainder of 35. This remainder is reduced to tomines by multiplying by 8, the result being 280. If we divide this 280 by 124, we have 2 tomines with a remainder of 32. Multiplying this 32 by 12, the number of grains in a tomin, we have 384, which divided by one hundred and twenty-four gives 3 grains.* The entire result now is 3485 pesos 2 tomines 3 grains, the amount of assayed silver or gold purchased with 4321 pesos 6 tomines of currency at 24 per cent. If you wish to prove this to be true, add the commission at 24 per 100, which is 836 pesos 3 tomines 9 grains, and you will see that it checks.

<table>
<thead>
<tr>
<th>4321 pesos 6 tomines currency</th>
<th>000</th>
</tr>
</thead>
<tbody>
<tr>
<td>432175</td>
<td>dividend</td>
</tr>
<tr>
<td>124</td>
<td>divisor</td>
</tr>
<tr>
<td>3485 pesos 2 tomines 3 grains assayed</td>
<td>06293</td>
</tr>
<tr>
<td>836 pesos 3 tomines 9 grains</td>
<td>17055</td>
</tr>
<tr>
<td>commission for assaying</td>
<td>432175</td>
</tr>
<tr>
<td>32 remainder</td>
<td>1</td>
</tr>
<tr>
<td>by 12 grains</td>
<td>022</td>
</tr>
<tr>
<td>are 384</td>
<td>384</td>
</tr>
<tr>
<td></td>
<td>124</td>
</tr>
</tbody>
</table>

* The result should be $3\frac{3}{11}$, but the fraction is rejected. This shows again how difficult it was to perform such operations to a high degree of precision without the aid of decimal fractions. Indeed, the use of such denominations as pesos, tomines, and maravedis was due solely to the necessity experienced by the ancients for avoiding fractions. For example, 6 tomines is merely a substitute for $\frac{3}{2}$ of a peso, or 0.75 of a peso.

† The method of division here shown is about the last stage of the medieval galley method which had been in use in Europe for a long time. By that method the figures were canceled out as soon as they had served their purpose. Evidently, however, the Mexican press had no canceled figures in their fonts, and hence they do not appear in this text.
Declaraclon de la regla pasada.

Nota que la regla pasada es verisimilmente la regla de tres y así como ella se funda por plata se puede fundar por otras muchas formas que bien podría decir yo tengo 2000 botijas de vino que las 1000, son 25 por 100 mayores que las otras 1000 compran medidas todas con ellas chicas de por la medida delas grandes. Demoando en quantas se bolucharán las 1000 chicas medidas por la medida delas grandes. Para ella y las semejantes se fundar la regla desta manera. Si 125 delas chicas se toman en 100 delas grandes en que se tornará 1000 delas chicas multiplicando las 1000 por 100 serán 1000000 que es lo mismo como veces que añadir adelante las dos cifras pues parte 100000 por 125 y venir te han 800 y en tantas se toomaran las 1000 botijas chicas medidas por la medida delas grandes: la peñua es que les echas tu interés a 25 por 100 y venir te han 200 como veías figurado.

<table>
<thead>
<tr>
<th>1000 botijas grandes</th>
<th>25 por 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplicación</td>
<td>1000</td>
</tr>
<tr>
<td>Multiplicador</td>
<td>100</td>
</tr>
<tr>
<td>El producto</td>
<td>1000000</td>
</tr>
<tr>
<td>Entero</td>
<td>125</td>
</tr>
<tr>
<td>Caudalimiento</td>
<td>800</td>
</tr>
<tr>
<td>Enteros</td>
<td>280</td>
</tr>
</tbody>
</table>

Hacer de pesos ducales y de ducales pesos muy en breve.

Si quisieres saber tantos pesos quatro ducales son, saca el quito de los pesos y sumalo conellos mismos y el remanente será lo que delesas saber.
Common Rules

Explanation of the former rule

Observe that the preceding rule is really the Rule of Three,* and in the same way that this applies to problems relating to silver it can be applied to as many other processes or kinds of problems as one may wish. For example, I have 2000 jugs of wine, 1000 are 25 per 100† larger than the other 1000. Buy them all from me so that you will get the small ones at a price proportional to the price of the large ones. First find how many of the small jugs are equivalent to 1000 of the large jugs. For this and for like problems the rule can be applied in this way: If 125 of the small ones are equal to 100 of the large ones, to how many large ones are the 1000 small ones equal? Multiply the 1000 by 100 and the result, 100,000, is the same as if you had annexed two ciphers; now divide by 125 and the quotient is 800, the equivalent of 1000 small jugs. To prove this, increase the number 800 by 25 per 100 and the result is 1000, as you see worked out below.

1000 large jugs, 25 per 100 more than 1000 small ones.

Multiplicand 1000
Multiplier 100
Product 100000
Divisor 125
Quotient 800
Percentage ‡ 200

A short method of changing pesos into ducats and ducats into pesos

If you wish to find how many ducats there are in a certain number of pesos, take one fifth of the number of pesos, add it to the number of pesos, and the result is what you wish to find.

* The Rule of Three was the most popular of all the medieval commercial rules. It came to Europe, through the Arabs, from the Hindu arithmeticians. In the Middle Ages it went by such names as the Regula de Tribus, Regula Rerum Trium, Regula Aurea, and Regula Mercatorum.

† The Latin form was 25 per centum, whence 25 por ciento, 25 %, 25%, 25, and 25%.

‡ It will be observed that there are two inaccuracies in the original edition, namely, 10000 appears for 1000, and 280 for 200, “ynteres.”
Reglas ordinarias.

**Ejemplo:**

C. 445, $4 el quint es. 89, que sumados juntos montan 534. Y tan
tos uucados son los. 445, $4.

C. 265, $4 el quint es. 53, sumados cólos. 265. son 318. como veys

tos,

C. Si quisieres saber una cantidad de uucados quantos pesos son
faca el seismo de los uucados, y lo que restare sera lo que buscas.

**Ejemplo.**

C. 534. uucados el seismo es. 89. restados de. 534. quedan. 445. y ta
tos pesos son los. 534. uucados.

C. 318. uucados: el seismo es. 53. restados de. 318. quedan. 264.

C. Reduzir pesos a maravedís sin multiplicar.

C. Si quisieres saber una orden muy buena para saber una cantidad
de pesos quantos maravedís son por muy mas facil y breue mane-
ra a multiplicar: haz los pesos millares y saca el diezmo, y velo que
restare la mitad y aquello sera lo que buscas saber.

**Ejemplo.**

C. Toma. 456, $4 haz los millares son. 456000. el diezmo de los
quales es. 45600. (como veys figurado) q restados del principal
quedá, 410400. la mitades es. 205200. maravedís y tanto montan los
456. pesos a razon de. 450. maravedís el peso.

456, $4. son 456000.
El diezmo es 45600.
Restan 410400.
La mitades 205200. maravedís.

C. Semijante ala paseada en mayor cantidad.

C. Toma. 34568. pesos. 4. tomínes y haz los millares y por el me-
dio peso pon. 500. serán. 34568500. el diezmo es. 3456850. re-
stantados del principal quedan. 3111650. la mitades es. 15555825. mara-
vedías como veys por la figura.
Common Rules

Example

One fifth of 445 pesos is equal to 89, which added to 445 gives 534, the number of ducats in 445 pesos.

One fifth of 265 pesos is 53. Adding this to 265 gives 318, as you see.

If you wish to find out how many pesos there are in a number of ducats, subtract one sixth of the number of ducats from the number of ducats.

Example

One sixth of 534 ducats is equal to 89, which subtracted from 534 is equal to 445, and this is the number of pesos in 534 ducats.

One sixth of 318 ducats is equal to 53, which subtracted from 318 gives 265.

To reduce pesos to maravedis without multiplying

If you wish a good rule for finding the number of maravedis in a certain number of pesos by a much easier and shorter method than that of multiplying, raise the number of pesos to thousands, find one tenth, subtract it from the number of thousands, and then find one half of the remainder; this gives the required number.

Example

Raise 456 pesos to thousands and you have 456,000; one tenth of this is equal to 45,600 (as you see by the work); this subtracted from the principal leaves 410,400, the half of which is 205,200 maravedis, and this is the number of maravedis in 456 pesos at the rate of 450 maravedis to a peso.

456 pesos are 456000.
The tenth is 45600.
The remainder 410400.
The half is 205200. maravedis.

Similar to the above, using larger figures

Take 34,568 pesos 4 tomines and raise them to thousands, and for the half peso put 500, and the result will be 34,568,500. One tenth of this is 3,456,850; and this subtracted from the principal leaves 31,111,650. One half of this is 15,555,825 maravedis, as you see by the work.
Reglas ordinarias. fo. cxxxiii

3 4 5 6 8, p. 4, to.

Cy o esta tengopozmo,

Jos é breve y certa co-

mo veras si lo ves.

El vien,

El viezmo. 3 4 5 6 8 5 0 0.

El viezmo. 3 4 5 6 8 5 0.

La mitades. 3 1 1 1 1 6 5 0.

La mitades. 1 5 5 5 8 2 5 8 5 0.

Si quisieres saber tantos pesos quantas coronas son sin hazer-

lo por maravedis, multiplica los pesos por nueve y parte por siete,

y el aduenimiento seran coronas.

Ejemplo.

156, p. 8, multiplica por 9, son 504, parte por 7, viene 72, coronas.

Para hazer otras coronas pesos, multiplica por 7, y parte por 9.

Ejemplo.

63, coronas, multiplica por 7, son 441, parte por 9, vienen 49

que son pesos por la suma; también se pode hazer desta manera, a-

los 56, pesos añade sus dos setenes que son 16, y seran las 72, coro-

nas, la otra diás 63, coronas resta sus dos nouencos que son 14, que
dan los 49, pesos como rega.

Para hazer de ducados coronas y de coronas ducados, aunque

esta puesto por quenta, multiplica los ducados por 15, y parte por

14, para hazer de coronas ducados, multiplica por 14, y parte por

quinze, y los últimos aduenimientos seran lo que busca.

Ejemplo.

42, ducados, multiplica por 15, son 630, parte por 14, y venirse

han 45, y tantas coronas son los 42, ducados.

Ejemplo.

60, coronas, multiplica por 14, son 840, parte por 15, vienen 56,

y tantos ducados son las 60, coronas; también lo puedes hazer co-

mo lo pasado; y es que los 42, ducados ajustes su catovanzo, que

estres: con que son las 45, coronas: así mismo alas 0, coronas.
Common Rules

34568 pesos 4 tomines
I consider this the shortest and best and surest method, as you will see if you use it.

If you wish to find how many crowns there are in a certain number of pesos, without reducing to maravedis, multiply the pesos by nine and divide by seven, and the quotient will be crowns.

Example
56 pesos multiplied by 9 is 504. Divide by 7 and the result is 72 crowns.
To reduce crowns to pesos, multiply by 7 and divide by 9.

Example
63 crowns multiplied by 7 is equal to 441. Divide this by 9 and the result is 49, which result is pesos. These two rules may be worked out in this way: For the first rule, to 56 pesos add its two sevenths, which is 16, and the result will be 72 crowns. For the second rule, from 63 crowns subtract its two ninths, which is 14, and the remainder is 49 pesos as you see.

To reduce any number of ducats to crowns, multiply the ducats by 15 and divide by 14; and to reduce crowns to ducats multiply by 14 and divide by 15. The quotients will be the desired numbers.

Example
42 ducats multiplied by 15 is equal to 630; divide this by 14 and the result is 45, and so many crowns are the 42 ducats.

Example
60 crowns multiplied by 14 is equal to 840; divide this by 15 and the result is 56, and so many ducats are the 60 crowns.

Also you may reduce the first of the above sums, involving the 42 ducats, in this way: add to 42 its one fourteenth, which is three, giving 45 crowns. Proceeding in a similar manner with the second, from the 60 crowns, since they are
Reglas ordinarias.

porque vienen a menos, restales su quinta que es, 4. quedá los 56. ducados.

Caso de memoria:

Si quisieres saber de memoria muy fácil y verísimamente una cantidad de maravedís, cantos pesos son, dobla los miles y del pliego añadeles su diezmo hasta que no haya decenas, y ésta su suma por cada unidad tome, o maravedís, y posqué mejor la entiendas nota este ejemplo porque me parece te bastará.

Ejemplo.

Casa 1.200.000, maravedís, dobla los miles son 2.400, el diezmo es 240, y de 2.400,9 todos son 2.666 pesos, y de los se 98300, maravedís, y tanto valen los 1.200.000, maravedís.

Caso para saber lo que se veue de quinto de qualsquier plata corriente que se suere a quintar.

Si fuere a quintar alguna plata corriente y quisieres saber por qué la pluma de cabeza lo que hában de llevar de quinto, toma el quarto de lo que fuere a quintar razonando el marco a ab. y después toma uno pozo, 100, o el mismo y sumalo con ello, y la su suma será lo que veues del quinto y derechos de uno pozo ciento y para más satisfacción tuya y de poper una figura.

Ejemplo.

Casa 4.575. 98 ba una raya por debajo y luego sale el quarto en 1143, pesos 6. tomes y después pon debajo uno por ciento de lo que suite a quinta que son 45, pesos 6. tomes pozo cuanto los 75, que te sobran son tres quártos de 100. y alí los tres quarteos de uno peso son 6. tomes lo cual suma con los de más y montaran 1189 pesos 4. tomes, y tanto es lo que te haban de llevar de quinto y derechos como veue figurado.
Common Rules

to be reduced, subtract the fifteenth part, which is 4, leaving 56 ducats.

A rule to be memorized

If you wish to memorize an easy and sure way for finding the number of pesos in a number of maravedis, double the number of thousands, then to the result add its tenth, and so on until there are no more tens. Then for every unit in the sum take 50 maravedis. In order that you may better understand this rule, consider the following example which seems to me to be sufficient.

Example

Take 120,000 maravedis, double the thousands, and the result is 240; the tenth of this number is 24, and the tenth of 24 is two, and the sum total is 266 pesos. In units column there is 6, and for each of these units take 50 maravedis and the result is 300 maravedis. Thus we have 266 pesos 300 maravedis, which is the value of 120,000 maravedis.

A rule for finding the tax on any amount of silver currency to be assessed

If you are to compute the tax on any amount of silver currency and wish to know how much they will demand, take one fourth of the amount to be assessed, reckoning the mark at 4 pesos; then take one per cent of the original amount, add the one fourth and one per 100, and the sum will be the tax and the one per cent fee demanded. For a better understanding I shall set forth an example.

Example

Take 4575 pesos, draw a line underneath, below this write one fourth of the number, or 1143 pesos 6 tomines, and below this write one per cent of what you took to be assessed, or 45 pesos 6 tomines. The 75 which is left over is three fourths of 100, and the three fourths of a peso is 6 tomines. Add these together and the sum, 1189 pesos 4 tomines, is what they will demand of your money, including the fee, as you will see in the work below.*

* That is, $25\% + 1\% = \frac{1}{4} + \frac{1}{100}$.
Mas de tener asis:  

§ si lo bizieres por mar

cos, que habes de tomar por cada marco, 1,59, y por cada onza 1,10, y por cada quarta tres granos, y mas el uno por ciento como teigo decho, y nota que todo lo del uno por ciento es los derechos del mar
cado; y no mas, poez aunq a el le viene el qrito mas de derecho que aq se le da su magenidad lo lleva de menos. Y la causa es que le paga el uno por ciento todo que le viene de quinto como tu velo lo lleva a quinar o por mejor dezir se lo quinta de balde.

C. Muy muchas maneras aq de multiplicar entre las quales yo teigo esta por la mejor y de mas verdad; certidumbre lo uno por no se llevar nada y memoria lo otro porque para la regla de tres no es necesario mudar las letras para auer de multiplicar.

C. Ejemplo.

Digo que multipliques 879, por 758, lo qual has de poner en la manera como ves figurado y dar entre la multiplicacion y multiplicado: una raya como esta. y dar debajo otra raya como aqui ves 875 \times 978, y luego col 8, primera letra de mano y quierdas que multiplicador multiplica todas las de la multiplicacion que son las de adelante dela raya poniendo las letras en ella manera, 8, 758, 9 son, 72, el siete que es vezena debajo del, 8, y el 2, que es unidad un grado adelante que esta debajo del, 7, y luego oscur una unidad a sus de poner la vezena en el mismo lugar debajo dela letra que tomasfe por multiplicado y por la unidad en el grado adelante un zero como este. y luego oscur otra vezena no a us de poner nada: pero la unidad en su lugar un grado adelante de do aula es estar la vezena y luego do 8, 7, 56, el 5, debajo del, 2, y el, 6, un grado adelante del, 2, y luego do 8, 7, 6, el 6, debajo del otro, 6, y el, 4, un grado adelante, que es debajo dela primera letra de la multiplicacion: ago
Common Rules

Let me advise that if you wish to do this by marks of weight, for each mark take 1 peso; for each ounce, one tomin; for each quarta, three grains, and besides the one per cent of which I have spoken.

Observe that the entire one per cent is the fee for the weigher or assayer, and no more, because, although there is due him one fourth more as a fee than is given him by his majesty, he receives less than this. The reason is that he pays to his majesty one per cent of that which is due himself of the tax, as you do on what you take to be taxed; or better expressed, it is taxed gratis.

There are many ways of multiplying,* among which I consider the following to be best and most accurate. For one reason, no memory work is required; and for another, according to the Rule of Three it is not necessary to move the figures in order to know how to multiply.

Example

To multiply 879 by 758, we must place the figures in order you see set forth, and draw between multiplicand and multiplier a line like this \, and then draw underneath them another line, thus: 875\978. Then, using the figure 8, the first one on the left-hand side, which is the multiplier, multiply all the figures in the multiplicand which are beyond the line in the following manner: 8 times 9 is 72; place the 7, which is tens, under 8, and the 2, which is units, under the next figure, which is 7. If you have no units, you must put the tens in their place under the first figure which you take as the multiplier, and in the place of units you must write a zero, like this: 0. If you have no tens, you must not put down anything, but must put the units in the column next to the one in which there would have been tens. Then observe that 8 times the 7 of the multiplicand is 56, and so we place the 5 under 2, and place 6 in the next column after 2. Similarly, 8 times 8 is 64, and we place the 6 under the other 6 and place 4 in the next column under the first figure of the multiplicand.

* The method most commonly used in the sixteenth century was not the one given in the example, but one of the two known by the Italian names gelosia and bericuocolo.
Reglas ordinarias.

ra vez el, 8, y tomo el, 7, de velaste con el cual 61, 7, veces nueve, 63, el 6, debajo del, 5, en la misma orden de el, 7, y el tres en grado adelante debajo del, 6, y luego de, 7, veces siete, 49, el quatro debajo del, 3, el 9, debajo del, 4, de adelante luego de, 7, veces 8, 86, el cinco debajo del, 7, siguiendo la letra de la multiplicación: agora vez el, 7, y toma el, 5, que es postrero multiplicado con el 7. del, 5, veces 9, 45, el, 4, debajo del otro, 4, en la orden del, 5, que tomas por multiplicado y el, 5, debajo del, 5, de adelante y luego de, 5, veces 7, 35, el, 3, debajo del, 5, y el, 5, debajo del, 6, y luego de, 5, veces 8, 40, el, 4, debajo del, 5, y el 3, itp, debajo de la postrera letra y luego suma.

C. Regla para cobrar de la magnitud por el quinto.

Muchos creen ay o auido y les deuo la magnitud dinero y para que no se haga un error es necesario buscar oro o plata por quitar para que por el quinto lo cobren los cuales lo puzo saber lo que an o llevar a quintar para cobrar lo que les deuo o llenar menos o de menos que el que lo que sea menos de más les es perjuicio en que la plata y oro que llenan las que la cueste interes y llenando de menos que es necesario bolver otra vez a quintar de que tengo entendido algunos reciben pesadumbre y para los tales con guda de Dios dar aqui una regla como facil y sin ningun erro sepan lo que ha de llevar a quintar para cobrar al justo lo que les deuo.

C. Regla.

Pon lo que se ve deo por suma ala cual añade dos zeros como estos, 00, y si quiere tomino pon por cada tomo en lugar delos zeros doce y medio como tengo dicho en la regla de cobrar plata corriente a en ayada y después parte aquella cantidad por veinte y seis y lo que ala particion salicre sera lo que delesas saber como aq vea.
Common Rules

Now leave the 8 and take 7 for the multiplier, thus: 7 times 9 is 63, so we place the 6 under 5 in the same column as the 7, and place the 3 in the next column under the 6. Then 7 times 7 is 49, and we place the 4 under the 3, and the 9 under the 4 in the column of the first figure of the multiplicand. Then 7 times 8 is 56, and we place the 5 under the 9, and the 6 under the 7 in the column of the second figure of the multiplicand. Now leave the 7 and take 5, the last figure of the multiplier. Then 5 times 9 is 45, and we place the 4 under the other 4 in the column of 5, the multiplier, and put the 5 under the other 5. Then 5 times 7 is 35, and we place the 3 under the 5, and the 5 under the 6. Then 5 times 8 is 40, and we place the 4 under the 5, and the zero under the last figure of the multiplicand, and then we add.

Rule for collecting what is due from his majesty

I believe there are and have been many to whom his majesty owes money. For those who have to collect, it is necessary for them to bring gold or silver to be assessed so that they may collect what is due. These people, because they do not know how much they have to take to be assessed in order to collect what is due them, take too much or too little. Taking too much is hurtful because commission is charged on the extra gold or silver that they take. Taking too little makes it necessary to assess again, from which, I understand, some unpleasantness results. For such as they, with the help of God, I will state here an easy rule, without any error, by which they may know how much to take to be assessed so as to collect justly what is due them.

Rule

Put down the sum that is due you; to it annex two zeros like these: 00. If you would like tomines, put for each tomin twelve and a half in place of the zeros, as I have stated in the rule for converting currency into assayed silver. Divide the sum by twenty-six, and the quotient will be what you wish to know, as you will now see.
Reglas ordinarias.

**Ejemplo.**

Digo que te dejan, 144, pesos a los cuales añade los dos ceros y montará, 114400, lo cual pre por 26 y viene ala partición, 4400, pesos que a 4, pesos el marco son, 1100, marcos y tantos has de llevar a cistar para cobrar los, 1144, pesos la prueba será que saques del \( \frac{1}{4} \) y despú es uno por ciento como te tengo di chgo en la segunda regla antes vista. De aquí adelante pondre algunas preguntas que aunque no están necesarias para lo que en este regio se versa los que son adicionados a la cuenta se harán con ellas porque aunque no son futilses para los que algo saben los que lo deseen saber con ellas tendrán puestos para más subir.

**I. Pregunta.**

Fui a quitar cierto oro no sé lo que pague del quinto por qué me quitaron de una libranza pero sé que sacados quintos a derechos me costó este oro sin interesse, 1584, pesos; diviendo quinto es lo que me quitaron en la libranza y qué es lo que agora vale este oro. Regla saca el \( \frac{1}{4} \) de, 1584, es, 396, suma los 60, 1584, son 1980, los cuales parte 499, el aduñimiento es veinte añi- dos de 1980, son, 2000, y tanto vale agora el oro, suma, 396, con, 20 son, 416, y tanto se quito en la libranza veríssimo.

**I. Segunda pregunta.**

Semejante ala pasada fui a quitar un pedazo de oro no sé lo que pesaba antes y quitar pero sé que me quitaron del, 416, pesos de derechos. Diviendo que lo que me ha de quedar. Regla multiplica 416, 499, son, 2080, parte los 499, vienen, 80, restados de, 2080, quedan, 2000, de los cuales saca el \( \frac{1}{4} \) es, 400, restados
Common Rules

Example

Suppose that there is due you 1144 pesos. Annex to this number two zeros, making 114000; divide this number by 26 and the result is 4400 pesos which, at 4 pesos to the mark, make 1100 marks, the amount you ought to take to collect the 1144 pesos. To prove this, find \( \frac{1}{4} \) of 4400, and then one per cent of 4400, as I have stated in the second rule preceding this one, and then add.

From now on I shall propose certain questions in which, although not necessary for what is used in this kingdom, those that like arithmetic will delight. Although they are not difficult for those who know something of mathematics, those who desire to know more will find in them the beginnings for further advance.

First problem

I took a certain amount of gold to have taxed. I do not know what tax I paid, because it was paid from a bill of exchange, but I know that, deducting the tax and the fees, this gold cost me without commission 1584 pesos. I want to know how much they took from me in the bill of exchange and what the gold is now worth.

Rule: Take \( \frac{1}{4} \) of 1584, which is 396; add this to 1584 and the result is 1980. Now dividing 1980 by 99 the quotient is 20, and this added to 1980 gives 2000, the present value of the gold. Adding 396 to 20 gives 416, and this is what was taken from the bill of exchange.*

Second problem

Similar to the above, I went to assess a piece of gold. I do not know what it weighed before being assessed but I know that they took 416 pesos as the fee. I want to know what should be left for me.

Rule: Multiplying 416 by 5 we have 2080; dividing this by 26 there results 80. This subtracted from 2080 gives 2000, from which find \( \frac{1}{5} \), which is 400;

*We have 25\% of 1584 = 396, tax; 125\% of 1584 = 1980, value of gold less 1\%. Hence the value is 2000. Then 2000 - 1980 = 20, fee, and 396 + 20 = 416, total payment.
Reglas ordinarias.

Ve, 2000, quedan, 1600, de los quales se suma uno por ciento son 26, repetidos 5, 1600, quedan, 1584, y tanto es lo que ahora vale el 20. Mas breue ajustad dos ceros como estos, 00, adelante de, 416, son, 41600 parte por 26, vienen, 1600, de los cuales se suma uno por 2,100, vienen 16, restados de, 1600 queda n 1584, por la suma.

Te cerca pregunta.

E compuesto diez varas de terciopelo menos, 20, pesos por 2,34, pesos y mas una vara de terciopelo, ó más a como costo la vara. Formula sumas los pesos, 20, y 2,34, son, 54, que sera tu particion resta de las, 10, varas la una que dice y mas quedan, 9, varas por las quales gastos, 54, vienen, y tanto es el precio de cada vara. Puede una, 10 varas 6, pesos hacen, 60, pesos menos, 20, pesos quedan, 40, y 36 que costaron, 34, pesos y mas una vara que son, 6, con que vale los mismos, 40, pesos.

Quarta pregunta.

E compuesto, 12, varas de vicho menos, 30, pesos por 2,98, y menos cuatro varas veniado a como costo la vara, nota esta que es muy breue y verissima suma los pesos, 30, y, 98, son, 128, suma las varas, 12 y 4, son, 16, pesos los quales parte, 128, venirete an, 8, y tanto es el precio de cada vara. Puede una, 12, varas 8, son, 96, menos 30, és, 66, dice costar 6, 98, menos, 4, varas que son, 32, pesos que como vengo, quedan los dichos, 66.

Quinta pregunta.

E compuesto, 9, varas de vicho por tanto mas de, 40, pesos, quanto, 13, varas al mismo precio vallen menos de, 70, veniado a como costo la vara. Formula has como ella passada suma los pesos, 40, y 70, son, 110, suma las varas, 9, y, 13, son, 22, pesos los quales parte los 110, el advenimiento es, y tanto es el precio de cada vara. Puede una, 9, varas 5, pesos son, 45, pesos que son, 5, mas de 40, y, 13, varas 5, son, 65, que son, 5, pesos menos de, 70, como vengo.
Common Rules

this subtracted from 2000 leaves 1600, of which find one per cent, which is 16; and this subtracted from 1600 gives 1584, the present value of the gold. Briefly, annex two zeros to 416, making 41,600; then divide by 26, giving 1600. Take one per 100 of this, or 16, and 1600 minus this gives 1584, the required sum.*

II Third problem

II I bought 10 varas of velvet at 20 pesos less than cost, for 34 pesos plus a vara of velvet. How much did it cost a vara?

Rule: Add 20 pesos to 34 pesos, making 54 pesos, which will be your dividend. Subtract one from 10 varas, leaving 9. Divide 54 by 9, giving 6, the price per vara.

Proof: 10 varas at 6 pesos is 60 pesos. This minus 20 pesos is 40. You paid 34 pesos plus a vara costing 6 pesos, and this gives the result, 40 pesos.†

II Fourth problem

II I bought 12 varas of velvet at 30 pesos less than cost, for 98 pesos minus 4 varas. How much was the cost per vara? The following is a short method: add the 30 pesos and the 98 pesos, making 128; add the number of varas, 12 and 4, making 16; divide 128 by 16, giving 8, the price per vara.

Proof: 12 varas at 8 pesos is 96 pesos; this less 30 pesos is 66. You paid 98 pesos minus 4 varas, or 32, and this leaves 66.‡

II Fifth problem

II I bought 9 varas of velvet for as much more than 40 pesos as 13 varas at the same price is less than 70 pesos. How much did a vara cost?

Rule: Add the pesos, 40 and 70, making 110. Add the varas, 9 and 13, making 22. Dividing 110 by 22 the quotient is 5, the price of each vara.

Proof: 9 varas at 5 pesos are 45 pesos, which is 5 more than 40 pesos; and 13 varas at 5 pesos are 65, which is 5 pesos less than 70, as you see.§

* The second method amounts to this:

\[ 416 = 1\% + 25\% \text{ of amount assessed} \]
\[ = 26\% \text{ of amount assessed.} \]
\[ \frac{100}{26} \times 416 = 1600, \text{ amount assessed.} \]
\[ 1\% \text{ of } 1600 = 16, \text{ fee.} \]
\[ \text{Therefore } 1584 \text{ is left after the fee is paid.} \]

† \[ 10x - 20 = 34 + x, \]
\[ x = 6. \]

‡ \[ 12x - 30 = 98 - 4x, \]
\[ x = 8. \]

§ \[ 9x - 40 = 70 - 13x, \]
\[ x = 5. \]
**Quadrados.**

9. varas por tanto mas dc. 40. ps. 40 9
quinto. 13. varas ab 70 ps. 70 22
mism. son menos. 1 10 00

**Quistiones por los numeros quadrados** 2 2

Para aver de hazer qualquier pregunta que te fuere omandada de numeros quadrados, es necesario que sepas que es numero quadrado y por que se llama quadrado, y que es numero cubo y porque se llama cubo.

Numeros quadrados se llaman y son aquellos que nacen de la multiplicacion o son produzidos de algun numero en otro semejante como 4, 9, 16, etc. descr. 4, nace del, 2, multiplicado por si mismo y sigue do, 2, vezes 2, son, 4, etc. 9, nace del, 3, por el mismo y sigue poque, 3, vezes 3, son, 9, de los cuales numeros los lineales como el, 2, 0 el, 3, son las rayzas;

Numeros cubicos se llaman y son aquellos que son contenidos de 3, numeros yiguales lineales dela qual multiplicacion son dichos ser procreados asi como, 8, 27, 64, 1 25, etc. porque, 8, nace de, 2, y del puede enel produzido es a saber, 2, vezes 2, 4, etc. 2, vezes 4, 8, 2, 3, vezes 3, 9, y, 3, vezes 9, 27, etc.

**Primera quistion.**

Dame un tal numero que bajustandole, 15, haga numero quadrado y restando del, 4, sea lo mismo, regla suma, 15, y, 4, son, 19, a-
justales, 1, son, 20, toma la mitad que es, 10, quadrados en si vizien-


Square Numbers

1. 9 varas for as much more than 40 pesos as 13 varas at the same price is less than 70 pesos.*

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Problems relating to square numbers

In order to know how to solve any problem that is given to you relating to square numbers it is necessary to know what a square number is and why it is called a square, and also what a cube number is and why it is called a cube.

A square number is a number that is derived by the multiplication of a number by itself, as is the case with 4, 9, 16, &c. The 4 comes from multiplying 2 by itself, as when we say that 2 times 2 is 4; and the 9 is the product of 3 multiplied by itself, because 3 times 3 is 9. Of such numbers the lineals like the 2 or the 3 are called the roots.†

A cube number is a number that contains the three identical numbers multiplied together, as is the case with 8, 27, 64, 125, &c.; for 8 is the product of 2 times 2 times 2; similarly, 3 times 3 times 3 are 27, &c.

First problem

Give me a number which, increased by 15, is a square number; and decreased by 4 is also a square number.‡

Rule for solving: Add 15 and 4, making 19; then add 1 to this result, making 20. Now take the half of this number 20, which is 10, and then square this result,

* The work here given shows the cumbersome method used in solving the equation

\[9x - 40 = 70 - 13x.\]

† In this work the word root, taken by itself, signifies square root.

‡ The rule depends on the fact that \(\frac{(a + b + 1)^2}{2} - a - b \text{ (or } a\) is a square.

37
**Cuestiones por los números.**

Do, 10, veces, 10, son, 100, de los cuales resta los, 15, que sean de aqui, sin arcto quedan, 85, y este es el número demandado, de el cual si restas los, 4, y da, 81, su raiz es, 9, 2, así mismo si le ajustas, 10, 15, son cierto raiz de los círculos 8, 10, porque, 10, veces, 10, son cierto y esto va para

C Segunda cuestion.

Dame un número que ajustándole, 8, sea cuadrado y restando el 8, quede cuadrado, toma medio de ocho es, 4, quade en es, 16, ajusta el, 1, es, 17, y este es el número demandando, al cual se ajusta, 8, hace, 25, que su raiz es, 5, y si le restas, 8, quedan, 9, que su raiz es, 3, porque, 3, veces, 3, son nueve como seye.

En mayor cantidad, da me un número que ajustándole, 20, sea número cuadrado y restando el, 20, quede cuadrado, toma mitad de 20, es, 10, quade en es, 100, ajusta uno hase, 101, y este es el un número que si le ajustas, 20, es cuadrado 2, si le restas, 20, queda cuadrado.

C Cuestion tercera.

Tiene uno dos restas muy buenas van le por ellas. 8, 8, no las quiere dar, viene uno cópialas como va en esta manera que le va para cada vara de cada una tantos tomínos quantas varas tiene aquella pieza, y hecha la cuenta no hallan que van parten que los 8, pesos que dan el primero, de modo que varas tenía cada una por si para la cual regla es mecheñer que busques dos números cuadrados tales, que juntos en uno no sean mas que uno quieras besar; y hagan número quadrado los cuáles números son, 3, y, 4, que multiplicados cada uno por si se mezclan siendo, 3, veces, 3, 9, 4, veces, 4, 16, y sumados el uno có el otro son, 25, y su raiz es, 5, y luego di por regla de 3, si cinco rayz de, 25, son venidos de, 8, que es el valor de...
Questions relating to numbers

thus: 10 times 10 is 100. From this subtract 15, and we have 85, and this is the number required, that is, the one from which if you subtract 4 you have 81, the root of which is 9. The same thing happens if you add the 15, the result being a hundred, the root of which is 10; for 10 times 10 is 100, which checks.

☐ Second problem

☐ Required a number which increased by 8 is a square, and decreased by 8 is also a square. Take half of eight, which is 4; square it, making 16; add 1, making 17, and this is the number which increased by 8 is 25, the root of which is 5; and which decreased by 8 is 9, the root of which is 3; for 3 times 3 is 9, as you see.*

☐ Using larger numbers, required a number which increased by 20 is a square number, and decreased by 20 is also a square number. Take half of 20, namely 10; square it, making 100; add one, making 101, and this is a number which increased by 20 is a square, and decreased by 20 is also a square.

☐ Third problem

☐ A man has two very good ropes for which he can get 8 pesos, but he refuses the offer. Someone offers to buy them by the vara in such a way that for every vara in each rope he gets as many tomines as there are varas in that rope. When the computation is made, they find that the money is no more than the 8 pesos which the first one offered. How many varas are there in each rope?

To solve this it is necessary to find two numbers whose squares added together make a number which is no greater than that of the one given. These numbers are 3 and 4. Multiply each number by itself and we have 3 times 3 which is 9, and 4 times 4 which is 16; and these added together are 25, and the root of this is 5. Then by the Rule of Three, as five, the square root of 25, is to 8, the value

*This depends on the fact that

\[ \frac{x^2}{4} + 1 + x = \left( \frac{x + 2}{2} \right)^2, \text{ a square}, \]

and

\[ \frac{x^2}{4} + 1 - x = \left( \frac{x - 2}{2} \right)^2, \text{ also a square}, \]

the rule simply giving the expression \( \frac{x^2}{4} + 1. \)
Quadrados. fo. xviiii.

lo que vendio adeo vendran, 3, 7, 4, que fueron los numeros hallados multiplica, 8, poz, 3, son, 24. parte poz cinco vienen quatro y 
3/4. y estas son las varas de la vna, luego vi, 8, vezes, 4, son, 32, parte poz, 5, vienen, 6, 1/2 y estas son las varas de la otra que sumadas entrambas tienen, 11, varas 2 un quinto 2 vendidas cada vna poz si bado poz cada vara de cada,1 tantos tomiones quitas varas tiene vienen a valer las, 8, 58, la prueba multiplica, 6 1/2. poz, 6, 2/3 vienen 40, 2 1/2 y multiplica, 4, 7, 4, quinto poz, 4, 1/4 vienen, 23, 2, 1/2 que sumados son, 64, tomiones 2 partidos poz, 8, tomiones que tiene un peso son, 8, pesos.

Questión quarta.

És que fuése pedida una questião en tal manera que se desessen, da me un numero quadrado y tal que quierand o vel una cantidad certa que de quadrado y ajustando se a lea quadrado, para aver se absolover una tal questião es necessário que sepan que cosa es numero congruo y que cosa es numero congruente.

Un numero congruo se llama y es un tal numero que se abto a dar y se cebir otro numero el cual se llama congruente en tal manera que dale o recibiendo lo siempre sea quadrado y para que mejor se mas claramente lo entiendes podere aqui bajo los numeros congruos y congruentes que me pareceran necessarios, y así mismo podre un exemplo, é que si bien teneras poderas declarar todas las questiões é que estas via se fue en demandadas siendo tal el numero desmandado é que balle numero congruo é partico poe el el aduenmié to sea numero quadrado, poz no lo escriendo agora secreto, el cual se lo por no ser proflito.
Square Numbers

at which it was sold, so we have 3 and 4 to the numbers to be found. Multiply 8 by 3 and we have 24; divide by five and we have four and \( \frac{4}{5} \), and this is the number of varas in one piece. Then 8 times 4 is 32; divide by 5 and we have 6, \( \frac{2}{5} \), and this is the number of varas in the other piece. These added together give 11 varas and a fifth. If we pay for each vara of each rope as many tomites as there are varas, the value comes to 8 pesos.

Proof: Multiply \( 6\frac{3}{5} \) by \( 6\frac{2}{5} \) and we have 40 and \( \frac{3}{25} \). Multiply 4 and 4 fifths by 4 and \( \frac{1}{5} \) and we have 23 and \( \frac{1}{5} \), which added together gives 64 tomites. This divided by 8, the tomites in a peso, gives 8 pesos.*

\( \square \) Fourth problem

Suppose that you were given this problem: Find a square number such that if we take from it a certain number, there remains a square; and if we add to it the same number it is also a square. In order to solve such a problem it is necessary to know the nature of a congruent number and of a congruent number.

A congruous number is such a square number that, subtracting from or adding to it another number, called a congruent number, it will still be a square. So that you may better and more clearly understand I set forth below the congruous and congruent numbers which I think necessary, and I also give an example by which, through careful examination, you will be able to solve all the problems of this kind that may be proposed. The number required must be such that when the congruent is divided by it the quotient will be a square; if it is not, there is another secret way, but this I will not give lest I be too prolix.†

* We have \( x^2 + y^2 = 8^2 \). If we take \( v = 3 \) and \( w = 4 \), we have \( v^2 + w^2 = 5^2 \). Hence the author assumes that \( 8 : 5 = x : 3 \), and that \( 8 : 5 = y : 4 \).

† This congruent number is what Leonardo Fibonacci (1225) called a congruum, a number of the form \( 4xy(x + y)(x - y) \). He gives the problem: "To find a number which, being added to or subtracted from a square number, leaves a square number," and uses the identities

\[
(x^2 + y^2)^2 - 4xy(x^2 - y^2) = (y^2 + 2xy - x^2)^2, \\
(x^2 + y^2)^2 + 4xy(x^2 - y^2) = (x^2 + 2xy - y^2)^2.
\]
### Quistiones por los numeros.

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<tr>
<th>CLogruos.</th>
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<th>CExemplo.</th>
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<tr>
<td>.25, Re.p da.</td>
<td>24.</td>
<td>Da me vn numero quadrado tal que ajustádole 6, haga numero quadrado y restando del 6, quede numero quadrado, para lo qual has de buscar vn tal numero congrue te que partiendole por 6, vega numero quadrado el cual como a fue ra vegs el pinero es 24, pues par te, 24, por 6, viene, 4, que es quadrado (y su raiz es dos) y luego to ma el numero congruo quadrado correspondiente deste numero con gruente que es 25, partale por los 4, que es el aducimiento de el numero vienen, 6, $\frac{1}{4}$ y aqueste es el numero vemiadedado que si le aju stas seys baze, 12, $\frac{1}{4}$ que es numero quadrado y su raiz es, 3, $\frac{1}{2}$ y si reitas del 6, queda $\frac{1}{4}$ y su raiz es $\frac{1}{2}$ porque media vez me dia es $\frac{1}{2}$ y el mismo es quadrado que su raiz es dos y medio. (\text{Ita in alia.})</td>
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Questions Relating to Numbers

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<th>Congruents</th>
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<td>25</td>
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<td>Find a square number which being increased by 6 will still be a square, and which being decreased by 6 will also be a square. To solve, you must find a congruent number which being divided by 6 the quotient will be a square. The first number, as you see, is 24; this divided by 6 gives 4, which is a square (and its root is two). Now take the congruous number corresponding to 24, which is 25. Divide it by 4, which is the quotient of the first one, 24, divided by 6, and we have 6, $\frac{1}{4}$, and this is the required number. Add 6 to it and you have 12, $\frac{1}{4}$, which is a square number, the root being 3, $\frac{1}{2}$. If you subtract 6 you have $\frac{1}{4}$ and the root is $\frac{1}{2}$, since a half times a half is $\frac{1}{4}$ and the same number, $6\frac{1}{4}$, is a square of which the root is two and a half. And so with others.*</td>
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* In the list read 1521 for 1212, 2601 for 2602, 3721 for 3221, 2925 for 2905, and 2016 for 2026. In the problem, since

\[25 \pm 24 = x^2,\]

we have

\[\frac{25}{4} \pm \frac{24}{4} = \frac{x^2}{4} = y^2.\]
Quadrados.

Quiesión quinta.

Si quieres ballar o te fuere demandado que busques tres números quadrados o más y tales que juntos en uno hagan número quadrado: toma el primer número quadrado impare, 9, el cual que ta uno quedan, 8, toma la mitad, quadrado son, 16, y esto es el segundo ajusta, 9, y, 16, son, 25, quita uno quedan, 24, toma la mitad es, 12, quadrado, son, 144, y este es terceros: si lo quieres ver suma, 9, y, 16, y 144, son, 169, rayz de los cuales es, 13, como ves: y nota á por esta via lo podras hazer in infinitum.

Quiesión sexta.

Digo que me ves que en tal número quadrado que quitándole osajándole sus tres rayzes hagan número quadrado. Reglas: ten a niétese un número congruente y anual el número pursuante quadrado correspondiente y en el tal número congruente parte le puestos sus unidades cantidad son las rayzes que manda ajustar o quitar y para el aduanamiento parte el número suyo es grue quadrado con congruente y el último aduanamiento quadrado lo en sí mismo y lo procteluto, será el número demandado como veras por este ejemplo.

Exemplo.

Toma, 24, primer número congruente parte le 601, 3, que son las rayzes demandadas viene, 8 por los cuales parte, 25 que es su grado quadrado correspondiente vienen, 3, 9, 3, 25 lo cual quita en sí mismo lo producto es, 9, y 84. Y esto es el número demandado á ajustádole o quitándole sus, 3, rayzes sera quadrado como veas ajustale, 9, y 24, que son las tres rayzes monta, 197, 64, que es número quadrado y su rayz es, 8, y 8 si le quitas sus, 3, rayzes que dan 54 que es número quadrado y su rayz es, 8, 5, 0 chabos.

Septima quiesión.

Nota esta, 2, vezas, 2, son, 4, 3, 3 rayzes, 3 son, 9, sumados juntos hazen, 13, pone dame otros, 2, números que no sean, 2, ni, 3, 7, que quadrados en sí mismos y lo producto sumado juto seá los mismos, 13.
Square Numbers

Fifth problem

If you wish to find three or more square numbers which added together make a square, take the first odd square number 9; subtract one, which gives 8; take the half, its square being 16, and this is the second number. The sum of 9 and 16 is 25; subtract one and we have 24; the half of 24 is 12; the square of 12 is 144, and this is the third number. If you wish a proof, the sum of 9 and 16 and 144 is 169, of which the root is 13, as you see. In this way, you can solve any number of problems.*

Sixth problem

Find such a square number that if you subtract from it or add to it thrice its root, you have a square number. Keep in mind some congruent number and its corresponding square congruous number. Divide the congruent number by the number by which you are to multiply its roots when you add or subtract them. Divide this quotient into the corresponding congruous square number and square the quotient. The result will be the required number, as you will see by the following example.

Example

Take 24, the first congruent number. Divide it by 3, the number by which you are to multiply the roots, and the quotient is 8. Divide this 8 into 25, the corresponding square congruous number, and the quotient is 3 and \( \frac{1}{8} \). The square of this is 9 and \( \frac{1}{64} \), the required number which, added to or subtracted from thrice its root, will be a square as you see. Add 9 and \( \frac{1}{64} \), which is thrice the root, and you have 19 and \( \frac{9}{64} \), which is a square whose root is 4 and \( \frac{9}{8} \). If you subtract thrice the root, you have \( \frac{25}{64} \), a square number whose root is \( \frac{5}{8} \).

Seventh problem

Observe this: 2 times 2 is 4, and 3 times 3 is 9; these numbers added together make 13. Now find two other numbers, neither 2 nor 3, which squared in the same way and added together will give the same result 13.

* Take as the first odd number \( 2n + 1 \). Following the directions, the first square is \( 4n^2 + 4n + 1 \), the second is \( 4n^4 + 8n^3 + 4n^2 \), and the third is the square of \( 2n^4 + 4n^3 + 4n^2 + 2n \).

† Since \( 25 - 24 = 1 \),
we have, multiplying by \( \frac{9}{4} \), \( 9\frac{9}{4} - 9\frac{9}{4} = \frac{9}{4} \), a square, and similarly for \( 25 + 24 = 49 \).
Regla.

Busca. 2. números. dados que hagan numero cuadrado que tenga raíz discreta los primeros son 3, 4, 5. son 9, 25 juntos 32. son 9, 25, 5. son 9, 25, 5, 4. son los propuestos 5, que es su raíz poniendo como veces figura do. y luego multiplica en cruz hiriendo. 3. veces. 3. son 9, 2, 8. veces. 4. son 3, ponidos a la mano derecha el uno debajo del otro e luego buehue a vez ir por arriba. 2. veces. 3, son 6. por abajo 3, veces. 4. 12. resta el menor del mayor que es 6, se 12. quedan. 6. que son las partes poniendo las raíces delos números propuestos: el advenimiento es 1, y este es el 3 número demado y luego suma 9, 2, 8, que son los productos de los que primero multiplicaste son 17, los cuales pares poniendo el 5, el advenimiento es 3 ½, y este es el segundo número de madero, la puesta de los que á eles quatro. 1 ½ en simismo es 1 ¼, 1 ¾ 2 ½ en si mismos son 11. 1¼, que sumados juntos como veces son los mismos 13.

Es uno que tiene cinco pesas con las cuales puede pesar desde un tomín hasta quince pesos y mas. Demando que es lo que pesa cada una Puerta: Primera, 1, to. La segunda, 3, to. La tercera, 1, 8, 1, to. La cuarta, 3, 8, 3, to. La quinta, 10, 8, 1, tomín
Questions Relating to Numbers

Rule

Find 2 numbers the sum of the squares of which will make a square number which has an integral root. The first numbers are 3 and 4, for their squares are 9 and 16, and these added together make 25, the root of which is 5. Observe that you have 5 numbers; the first are 2 and 3; the next are 3 and 4, the proposed numbers; and there is also 5, which is their root. Place these numbers as you see in the figure below. Then use cross multiplication, saying "3 times 3 is 9, and 2 times 4 is 8." Place these numbers at the right-hand side, one under the other. Then multiply again at the top, 2 times 3 is 6; and underneath, 3 times 4 is 12. Now subtract the less from the greater, that is, 6 from 12, and there remains 6. Divide this by 5, the root of the assumed numbers, and the quotient is 1.\(\frac{6}{5}\), one of the numbers required. Now add 8 and 9, the products of the first multiplication, and the sum is 17. Divide this by 5 and the quotient is 3\(\frac{2}{5}\), and this is the second required number.

Proof: The square of 1\(\frac{6}{5}\) is 1\(\frac{9}{5}\); the square of 3\(\frac{2}{5}\) is 11\(\frac{3}{5}\); and these added together, as you see, make 13.*

\[
\begin{array}{cccccc}
5 & 2 & 6 & 3 & 9 & 17 \\
3 & 4 & 8 & 12 & & \\
\end{array}
\]

\[
\begin{array}{cccccc}
02 & 1 & 17 & 3\frac{2}{5} & 6 & 1\frac{3}{5} \\
5 & 5 & & & & \\
\end{array}
\]

A man has five weights with which he can weigh from 1 tomin to fifteen or more pesos. What is the weight of each? The first, 1 tomin; the second, 3 tomines; the third, 1 peso 1 tomin; the fourth, 3 pesos 3 tomines; the fifth, 10 pesos 1 tomin.†

* The equation \(u^2 + \nu^2 = 13\) is indeterminate. It is given by Diophantus (II, 9), a late Greek algebraist of c. A.D. 275. In Sir Thomas Little Heath's edition of Diophantus, second edition, page 145, Euler's general solution is given as well as the special solution leading to Diophantus's results. In the latter solution \((x + 2)^2 + (2x - 3)^2 = 13\), whence \(x = \frac{5}{8}\) and the two numbers are \(\frac{13}{8}\) and \(\frac{1}{8}\). This solution is rather more simple than the one in the text.

† This is the well-known Problem of the Weights. Expressed in tomines the weights are 1, 3, 9, 27, 81, a geometric progression. This solution requires that the weights be placed on either or both pans of the scales. It is evidently inserted to fill the page, having no close connection with the problems which immediately precede or follow.

47
Cuarto quistión.

T. Veeses 3, son 9, 2.4, veces 4, son 16, sumados 52, 25, que es número quadrado y su raíz es 5, y á los otros, dos números, que sean 3, 5, 4, y que quadrados son los mismos y lo producido sumado sea 25.

C. Regla.

C. Buscan dos números que juntos los quadrados en uno hagan número quadrado que tenga raíz discreta: toma 5 y ve a que sus quadrados son 25, 2.1, 4.4, que son 169, raíz de los cuales es 13, pues nota que en la pasada a ella semejante tuiiste, 5, números y aquí tienes 4, la causa es que 3, 5, 4, primeros números tienen raíz discreta que es 5, el cual es el uno vulgar. 4, y sirva poez el 3, 5, y el 4, de quién es raíz y los otros 10, 5, 12, números hallados o propuestos y su raíz que es 13, los cuales pon en la manera que veas figurado y con el 5, raíz de 3, 5, 4, primeros números mul. el 5, y el 12, números propuestos eligiendo 5, veexes 5, son 25, 2. 5, veexes veexes son 60 y parte entramos productos poez 13 que es la raíz de los números propuestos y el aduenamiento serán los números que buscas: parte 25, poez 13, vienen 1 12 11 que es el uno parte, 60, poez 13 vienen 4, 8 11 que es el otro los cuales si los multiplicas cada uno poez sí mismo y lo producido sumas serán los 13, que demandas como veas múlt. en 1 11 es 3, 12 169, multiplicar en si 4, 9 11 es 21, 100 suma los y son 25, como veas poez la figura.
Square Numbers

Eighth problem

3 times 3 is 9, 4 times 4 is 16, and the sum of 9 and 16 is 25, which is a square number having 5 for its root. Find two other numbers the sum of whose squares is 25.

Rule

Find 2 numbers whose squares added together will make a square having an integral root. Take 5 and twelve whose squares are 25 and 144; these added together make 169, the root of which is 13. Observe that in the previous problem similar to this one you had 5 numbers. Here you have 4. The reason is that 3 and 4, the first numbers, have an integral root which is 5, which is one of the 4 numbers here, and serves for the 3 and 4 of which it is the root; the others are 5 and 12, the assumed or presupposed numbers, and their root is 13. Put them down as you see below; then taking the root derived from 3 and 4, the first numbers, say "5 times 5 is 25, and 5 times 12 is 60"; divide each product by 13, which is the root of the sum of the squares of the proposed numbers, and the quotients will be the required numbers. Dividing 25 by 13, the result is \(1\frac{1}{13}\), which is the one part; dividing 60 by 13, the result is \(4, \frac{8}{13}\); square each quotient and add the results and the sum will be 25 as required. That is, \(1\frac{1}{13}\) squared is 3. \(\frac{11}{169}\); 4, \(\frac{8}{13}\) squared is \(21, \frac{51}{169}\); and their sum is 25, as you see by the work.*

\[
\begin{array}{c|c|c|c}
25 & 12 & 28 \\
5 & 13 & \text{Divisor} \\
12 & 13 & 13 \\
60 & & \\
\end{array}
\]

* This is simply a variant of the seventh problem. Euler's general solution, referred to on page 47, applies to the equation \(x^2 + y^2 = f^2 + g^2\). If \(f = 3\) and \(g = 4\), as in this case, the solution is

\[
x = \frac{8pq + 3(q^2 - p^2)}{p^2 + q^2}, \quad y = \frac{6pq + 4(p^2 - q^2)}{p^2 + q^2},
\]

where \(p\) and \(q\) may have any values whatsoever.
Notables. Quistiones.

Quistiones del arte mayor tocantes al algebra.

Primer a quistion.
Da me vn numero quadrado que restando del 15, y \( \frac{3}{4} \) quede su propia raiz.

Regra.
Digo que el numero sea vn a a cosa vemediala es media cosa multiplica la en fi baze \( \frac{1}{2} \) decenso ajunta lo a. 15, y \( \frac{1}{4} \) baze. 16, cuya raiz quadrada y mas el medio dela cosa es raiz del numero demaidado. Duen a qua raiz quadrada de 16, y mas el medio dela cosa que es: \( \frac{1}{2} \) y medio baze. 20, y \( \frac{1}{4} \) que es el numero quadrado demandado restavel. 15, y \( \frac{1}{4} \) quedan. 14, y \( \frac{1}{4} \) que es la raiz del propio.

Segunda quistion.
Es uno que se flet a en vn nauto y pregunta al maestre que es lo que ha se var de flet a el maestre vise que no le ha de lleu ar mas que los otros boluiendo el pasia jo a replicar quanto seria el maestre responde que han de ser tantos pesos que multiplicados por se y a puntando los alo producto el remanente sera, 1260, demando quanto vemandma el maestre.

Regra.
Digo que el flete sea vn a cosa de \( \frac{1}{2} \) a la mitad es media cosa quadrada en fi baze \( \frac{1}{4} \) decenso ajunta lo a. 1260, baze. 1260, y \( \frac{1}{4} \) a quarto son \( \frac{1}{4} \) to raiz delos que menos medio dela cosa es el numero demaidado del flete reduc a. 1260, y \( \frac{1}{4} \) a quarto son \( \frac{1}{4} \) a la raiz es. 71, medios rest a el medio dela cosa que es medio quedan. 70, medios que son, \( \frac{3}{4} \), \( \frac{3}{4} \) y tanto es lo que demaida del flete: Duen a multiplica. 35. en fi baze. 1225, ajunta los 35, son. 1260, y \( \frac{1}{4} \) es numero demaidado.

Tercero quistion.
Uno vende cabras no se las que son mas de que llego vn mercha re y le pregunta quantas abra el vendedor responde son tantas que si las multiplica en si y lo producto quadruplica el ultimo produci do sera. 90000, demando quantas cabras tenia.
Noteworthy Problems

Problems of the Arte Mayor, relating to algebra

First problem

Find a square from which if \(15\frac{3}{4}\) is subtracted the result is its own root.

Rule

Let the number be \(cosa\) (x). The square of half a \(cosa\) is equal to \(\frac{1}{4}\) of a \(zenso\) \((x^2)\). Adding 15 and \(\frac{3}{4}\) to \(\frac{1}{4}\) makes 16, of which the root is 4, and this plus \(\frac{1}{2}\) is the root of the required number.*

Proof: Square the square root of 16 plus half a \(cosa\), which is four and a half, giving 20 and \(\frac{1}{4}\), which is the square number required. From 20\(\frac{1}{4}\) subtract 15 and \(\frac{3}{4}\) and you have 4 and \(\frac{1}{2}\), which is the root of the number itself.

Second problem

A man takes passage in a ship and asks the master what he has to pay. The master says that it will not be any more than for the others. When the passenger again asks how much it will be, the master replies: “It will be the number of pesos which, multiplied by itself and added to the number, will give 1260.” Required to know how much the master asked.†

Rule

Let the price be a \(cosa\) of pesos. Then half of a \(cosa\) squared makes \(\frac{1}{4}\) of a \(zenso\), and this added to 1260 makes 1260 and a quarter, the root of which less \(\frac{1}{2}\) of a \(cosa\) is the number required. Reduce 1260 and \(\frac{1}{4}\) to fourths; this is equal to \(\frac{504}{4}\), the root of which is 71 halves; subtract from it half of a \(cosa\) and there remains 70 halves, which is equal to 35 pesos, and this is what was asked for the passage.

Proof: Multiply 35 by itself and you have 1225; adding to it 35, you have 1260, the required number.

Third problem

A man is selling goats. The number is unknown except that it is stated that a merchant asked how many there were and the seller replied: “There are so many that, the number being squared and the product quadrupled, the result will be 90,000.” Required to know how many goats he had.‡

* \(x^2 - 15\frac{3}{4} = x, x^2 - x + \frac{1}{4} = 16, x = 4\frac{1}{2}\), the negative root being neglected. \(cosa\) (thing) was the unknown \((x)\), and \(zenso\) (Latin \(census\)) was our \(x^2\).
† \(x^2 + x = 1260\), whence \(x = 35\).
‡ \(4x^2 = 90,000\), whence \(x = 150\).
Del arte. Mayor.

C Regla.
Digo que tenga una cosa de cabras multipli ca en si bas en 2 10 multipli ca el cen so po se 4, su es quadruplo bas en 4, sen so es igual a, 900 00, cabras y es numero parte numero po se cen so el adue nimiento es, 22 250, ray 3 de los tales son las cabras que tenia. Deue ma tom a, 150, ray 3 de, 22 250, multipli ca en si bas en, 22 250, multi pli ca los po se 4, que es quadruplallo son, 90 000.

C Quarta quifion.
Uno va por un camino pregunta a otro que leguas aula hasta na cierta parte el otro le responde ay tantas leguas que si las multi plica ya en si y lo producto partes po se 5, el aduenimiento sera, 80, de mandado que leguas abra en lo que vize.

C Regla.
Digo que ay a una cosa de legua quadra la en si bas en céso parte le po se 5, el aduenimiento es, 1/2, de cen so igual a, 80, leguas part e numero po se cen so que es, 80, po se, 1/2, e), aduenimiento es, 400, el aduenimiento es, 400, en si y es, 20, y lo producido parte po se 5, el aduenimiento sera, 80, numero deman dado.

C Quinta quifion.
Uno compra ropa dela tierra en tripla proporcion de tal fuerte q multipli cando el triple pse el quarto del fu triple que son las piezas de ropa que compio lo producto sera, 48, pesos demando que piezas de ropa compio.

C Regla.
Digo que compio una cosa de piezas de ropa po tres cosas de pe se que es en tripla proporcion de ropa multipli ca en quarto te cosa fa de pieza de ropa po se 3, costas de pesos es, 1/4, de cen so iguales a 48, pesos que es numero parte numero po se céso que es, 48, po se, 1/4, el aduenimiento es, 64, ray 3 de los cuales son las piezas de ropa que
On Algebra

Rule

Let a *cosa* represent the number of goats. Squaring this we have a *zenso*; multiplying the *zenso* by 4, which is a quadruple, makes 4 *zensos*, which is equal to 90,000 goats. Divide 90,000 by the number of *zensos*, and the quotient is 22,500 [not 22,250 as given], the root of which is the number of goats he had.

Proof: Square 150, the root of 22,500, and you have 22,500; multiply this by 4, which is quadrupling it, and you have 90,000.

Fourth problem

A man traveling on a road asks another how many leagues it is to a certain place. The other replies: "There are so many leagues that, squaring the number and dividing the product by 5, the quotient will be 80." Required to know the number of leagues.*

Rule

Let a *cosa* represent the number of leagues. This squared makes a *zenso*; and this divided by 5 equals \( \frac{1}{3} \) of a *zenso*, which is equal to 80. Divide 80 by \( \frac{1}{3} \) and the quotient is 400, whose root is the number of leagues required.

Proof: Multiply the root of 400, which is 20, by itself. Then divide the product by 5 and the quotient is 80, the number required.

Fifth problem

A man buys a number of pieces of clothing for three sums of pesos which are in triple proportion, so that multiplying the triple of the first by \( \frac{1}{3} \) of the number tripled,† which is the number of pieces of clothing, the product will be 48 pesos. Required to know how many pieces of clothing he bought.‡

Rule

Let *cosa* of pieces of clothing be bought for three *cosas* of pesos in triple proportion to the pieces of clothing. Multiply a quarter of the number of pieces of clothing by 3 *cosas* of pesos and you have \( \frac{1}{3} \) of a *zenso*, equal to 48 pesos. Divide 48 by \( \frac{1}{3} \) and the quotient is 64, the root of which is the number of pieces of clothing.

\[ * \frac{1}{3} x^2 = 80, \text{ whence } x = 20. \]
\[ † \text{That is, } 3 x \times \frac{1}{3} x. \]
\[ ‡ \frac{3}{4} x^2 = 48, x = 8. \]
Quistiones.

ezemplo costaron le el triple que es ra3 de 676, prueba multipli-
car ra3 de 676, que es, 24, por 2, que es el $\frac{1}{3}$ del su triple de ra
23 de 54, el aduenimiento es 48, numero demandado.

1. Setima quistion.

Anot tiene yeguas y vacas en quincupla proporcion de tal suerte
que si multiplicas las yeguas en si y las vacas en si y lo producto 
mas fueran 1694, demando quitas las yeguas y quantas las vacas.


Regla.

Digo que tenga una cosa de yeguas 5, cosas de vacas multipli-
pica una cosa en si baze un censo mul. 5, cosas en si baze 25, ceros
ayunta lo. Son 26. censos iguales a, 1664, yeguas y vacas nume-
ro demandado parte numero por censo que es, 1664, por 26, el ad-
uenimiento es, 64, cuya ra3 son las yeguas y el quincuplo las va-
ca es ra3 de 1600. Prueba toma ra3 de, 64, es, 8, quincupla
los baze, 40, que es ra3 de 1600, suma el quadrado de, 8, que son
las yeguas, con el quadrado de 40, que son las vacas baze, 1664,
que es lo demandado.

2. Setima quistion.

Anot tiene tres joyas & qudrupla proporcion de valor de tal manie-
r que multiplicando lo que vale la primera por el valor, dela segun-
da y lo producido por el valor dela tercera el ultimo producto sera,
1728, demando que es el valor de cada joya.

Regla.

Digo que la primera valga una cosa y la segunda, 4, cosas y la ter-
cera, 16, que como vesp estan en quadrupla proporcion mul una co-
sa por 4, cosas es, 4, censos multiplica los por 16, cosas dela terce-
ra baze, 64, cubos iguales a, 1728, que es numero parte numero
por cubo el aduenimiento es, 27, cuya ra3 cuba que es, 3, es el va-
lor dela primera y la segunda vale ra3 cuba de, 1728, que es, 12, y la
Problems

that he bought. They cost him three times this, which is the root of 676 [576].

Proof: Multiply the root of 676 [576], which is 24, by 2, which is \(\frac{1}{3}\) of its
cube, that is, of 8, the square root of 64. The result is 48, the number required.

\(\text{Sixth problem}\)

\(\text{A man has mares and cows in quintuple proportion, in such a way that if}
\)you square the number of mares and square the number of cows, the products
added will be 1664. Required the number of mares and the number of cows.*

\(\text{Rule}\)

\(\text{Let there be a cosa of mares and 5 cosas of cows. Squaring the first makes a}
\)zenso, and 5 cosas squared makes 25 zensos, and the sum is 1664 mares and cows,
the required number. Divide this number by the number of zensos, that is, divide
1664 by 26. The quotient is 64, whose root is the number of mares, and the
quintuple, or square root of 1600, is the number of cows.

Proof: Take the square root of 64; it is 8. Quintuple it and you have 40,
which is the square root of 1600. Add the square of 8, the number of mares,
to the square of 40, the number of cows, and you have 1664, which was required.

\(\text{Seventh problem}\)

\(\text{A man has jewels in quadruple proportion of value such that, multiplying the}
\)value of the first by the value of the second and the product by the value of the
third, the last product will be 1728. Required the value of each jewel.†

\(\text{Rule}\)

\(\text{Let the value of the first be one cosa; that of the second, 4 cosas; and that}
\)of the third, 16 cosas, which you see are in quadruple proportion. Multiply one cosa
by 4 cosas and this is equal to 4 zensos. Multiply this by 16 cosas and the result
is 64 cubes, and this is equal to 1728. Divide this number (that is, 1728) by the
cube (that is, by 64) and the quotient is 27, whose cube root is 3, the value of
the first jewel. The second one is worth the cube root of 1728, or 12; and the

\(\text{* } x^2 + 25x^2 = 1664 \text{ (not 1694 as in the original), whence } x = 8.\)
\(\text{† Take } x, 4x, 16x. \text{ Then } 64x^3 = 1728, \text{ whence } x = 3.\)
Del arte. Mayor.  

tercera vale, 48, que es rayz cuba de, 2304. Prueba mul. el valor de la primera que es, 3, por el de la segunda que es doze y el aduenimiento por la tercera, que es 48, lo producto delas multiplicaciones sera 1728.

8 Octava question.

C Tengo hijos y hijas en proporcio sís que altera de tal arte ¾ mul. los hijos por las hijas y lo producto por la mitad de los hijos el ultimo producido sera, 162, demando quantos son los hijos y quantas las hijas.

C Regla.

C Digo que los hijos sean una cosa y las hijas una cosa y media que es en sis que altera proporción mul. una cosa por una cosa y media es un censo y medio el cual multiplica por media cosa que es mitad de los hijos yajes ¼ de cubo yguales a, 162, hijos y hijas que es numero parte numero por cubo que es, 162, por ¼ el eduenimiento ento es, 216, rayz cuba de los que son los hijos y las hijas rayz cuba de, 729, Prueba mul. rayz cuba de, 216, que es, 6, por rayz cuba de 729, que es 9, hazen, 54, los cuales mul por medio de seis que son los hijos lo producto es, 162, que es el numero demandado.

C Novena question.

C Uno ha de hacer dos pagamentos equadupla proporción de meses en tal manera; que cuadrando el su cuadruplo y lo que saliere mul. por el cuadruplo y lo producido cubicando el ultimo producido sea 32768, demando a quantos meses son los pagamentos.

C Regla.
third is worth 48, the cube (sic) root of 2304.

Proof: Multiply the value of the first, which is 3, by that of the second, which is twelve, and this product by the value of the third, which is 48, and the product of the multiplications is 1728.

Eighth problem

A man has a certain number of sons and daughters in altera proportion such that multiplying the number of sons by the number of daughters and the product by half of the number of sons, the last product will be 162. Required the number of sons and the number of daughters.*

Rule

Let the number of sons be a cosa, and the number of daughters be a cosa and a half, which numbers are in altera proportion. Multiply one cosa by a cosa and a half and the result is a zenso and a half, which multiplied by half a cosa, which is half the number of sons, makes \( \frac{3}{4} \) of a cube which is equal to 162, the number of sons and daughters. Divide this number by the number of cubes, that is, divide 162 by \( \frac{3}{4} \), and the quotient is 216, the cube root of which is the number of sons, and the cube root of 729 is the number of daughters.

Proof: Multiply the cube root of 216, which is 6, by the cube root of 729, which is 9, and the result is 54; multiply this by half of six, which is the number of sons, and the result is 162, the number required.

Ninth problem

A man has two payments to make, in quadruple proportion of months, so that squaring the first, multiplying the product by the quadruple, and cubing this product, the result will be 32,768. Required to know how the payments were made.†

* By the ancient Greek theory of proportion (ratio), \( \frac{2}{3} x \) and \( x \) are in altera proportion. The word proportion was commonly used for ratio in the sixteenth century, and the same is still the case outside the school. Take \( x \) and \( \frac{2}{3} x \). Then \( \frac{2}{3} x^3 = 162 \), whence \( x = 6 \).

† \( 64x^9 = 32,768 \), whence \( x = 2 \).
Notables Quistiones.

TRegla.

Digo que los 2 pagamentos sean una cosa y 4 cosas que es cuadrupla propozicion cual sev2 su cuadruplo es un censo multiplica le po el cuadruplo es 4. cubos cubicalos ayes.64.cubos de cubos 
iguales a. 32768. que es numero; corte numero por cubos de 
cubos que es. 32768. por. 64. el aduentimiento es.512.cuya ra-
y3 cuba de ra3 cuba seran los meses del primer pagameto y ra3 
cubica de ra3 quadrada de.26214.4. sera el segundo pagameto. Don 
una ra3 cubica de.512.es.8.y ra3 cubica de.8.es.2. que es el pri-
mer plazo;2 ra3 quadrada de.26214.4.es.512.y ra3 cubica de.512. 
es. 8. que es el segundo plazo;quadr el. 2.q es su cuadruplo de.8. 
es.4. el qual multiplica por el cuadruplo es.32. cubicalos el ultimo 
produzido es.32768.número demandado.

TDecena quistion.

Un hombre tiene veo hijos en propozicion fis que quarta a bedad 
en tal manera que, mul.el su cuadruplo dela beded del menor por el 
su quintuplo dela bedad del mayor y lo que saliere quadruplando y 
veo produzido sacado su ra3 y cubicado la mitad el ultimo produ 
zido sera.125. años; demando que bedad tiene cada uno.

TRegla.

Digo que el menor ay una cosa de años y el mayor ay una 
cosa y. $\frac{1}{4}$ de cosa de años que es fis que quart propozicion, mul.el su 
quadruplo del menor por el su quintuplo del mayor que es. $\frac{1}{4}$ por 
$\frac{1}{4}$ lo producto es. $\frac{1}{16}$ de censo quadrupla. $\frac{1}{16}$ es. $\frac{1}{4}$ de censo 
su ra3 es media cosa cubica su mitad que es un qarto de cosa el vti 
mo produzido es. $\frac{1}{64}$ de cubo igual a.125. que es lo que se busca pre 
numero por cubo que es.125. por. $\frac{1}{54}$ el aduenimiento es. 8000, 
cuy ra3 cubica son los años del menor y ra3 quadrada de.625. 
son los del mayor.9. por un ma quema toma ra3 cuba de.8000.bedad del me 
no es.20. mul.el su quadruplo que es.5 por el su quintuplo de.25. 
ra3 quadrada de.625.que es.5. lo producto es.25. quadruplalos ha
Noteworthy Problems

曈 Rule
曈 Let the 2 payments be represented respectively by one cosa and 4 cosas, which are in quadruple proportion. Square the first, making a zenso; multiply this by the quadruple, giving 4 cubes; cube this, making 64 cubes of cubes equal to 32,768. Divide 32,768 by 64 and the quotient is 512, of which the cube root of the cube root is the number of months for the first payment. The cube root of the square root of 262,144 will be the number of months for the second payment.

Proof: The cube root of 512 is 8, and the cube root of 8 is 2, which is the number of months for the first payment. The root of 262,144 is 512, and the cube root of 512 is 8, which is the second number of months. Square the 2, which is a fourth of 8, and this is equal to 4, which, multiplied by the quadruple 8, is equal to 32. Cube 32, and the result is 32,768, the required number.

曈 Tenth problem
曈 A man has two sons whose ages are in such a quarta proportion that, multiplying one fourth of the age of the younger by one fifth of the age of the elder, and quadrupling the result, and cubing half the root, the final product will be 125 years. What is the age of each?*

曈 Rule
曈 Let the age of the younger be a cosa of years and that of the elder be a cosa and \( \frac{1}{4} \) of a cosa of years, which is a quarta proportion. Multiply a fourth of the younger by a fifth of the elder, which is \( \frac{1}{4} \) by \( \frac{1}{5} \). The product is \( \frac{1}{16} \) of a zenso. Now quadruple \( \frac{1}{16} \), and this is equal to \( \frac{1}{4} \) of a zenso, of which the root is half a cosa. Cube half of this and the product, \( \frac{1}{8} \) of a cube, is equal to 125. Divide 125 by \( \frac{1}{8} \) and we have 8000, whose cube root is the number of years in the age of the younger, and the square root of 625 is the number of years in the age of the elder.

Proof: The cube root of 8000 is 20, the number of years in the age of the younger; multiply a fourth of it, or 5, by a fifth of 25, the square root of 625, which is 5, and the product is 25; and this multiplied by 4 (that is, 25 being quadrupled)

* He means to multiply by \( \frac{1}{4} \) instead of 4, and \( \frac{1}{5} \) instead of 5. That is, the age of the younger is \( \frac{5x}{4} \); of the elder, \( \frac{5x}{4} \). Then \( \frac{5x}{4} \times \frac{5x}{5} = \frac{25x^2}{20} \); \( \frac{25x^2}{20} \times \frac{25x^2}{20} = \frac{625x^4}{400} \); \( \sqrt{\frac{625x^4}{400}} = \frac{x^2}{2} \); \( \frac{1}{2} \cdot \frac{x^2}{2} = \frac{x^3}{4} \); \( \frac{x^3}{4} = 125 \), whence \( x = 20 \).
De arte mayor. fo. ciii.

cen ciento, toma la mitad de su raíz es 25, cubícalos el último producido es 125. número demandado lo cual nota.

No he querido ser en esto más largo lo uno por evitar proximidad y lo otro porque como siempre he dicho mi intento nunca fue otro que poner las cosas necesarias en el común vestos reynos: y así verás como en lo de más he sido breve e suplico os, sea tomado e ser uicio y recibida la voluntad como de quien desea servir.
On Algebra

makes a hundred. Take half of the root of 100, which is 5; cube this, and you have 125, the number required, which note carefully.

It has not been my desire to extend this work, one reason being that I would avoid becoming tiresome, and the other reason being, as I have always said, that I have wished merely to set down the things which are necessary and familiar in this kingdom. You will therefore see that I have always written succinctly, and so I beg that this book may be judged and received merely as the work of one who seeks to be of service to his fellows.
Fin de la obra.

A honra y gloria de nso señor Jesu

Christo y de la bendita y gloriosa virgé santa Maria su madre
y señora nra. Al se acabó el presente tratado Intitulado Su
mario cópëndio de cuetas de plata y oro necesarias en
los reynos del Piru, El qual fue impreso en la muy
grande y lujosa y muy lical ciudad de México, en
casa de Juan pablo de Bessiano: con licencia del
muy Illustrísimo señor Don Luys de Te
lasco, Alzórez y gobernador desta Nueva
española, El mismo cuya licencia del muy
Illustr y reverenciisimo, S.Ob. fra
Alóso de Blotus archibispo de
mexico por cinto fue visto y exa-
minado, y se halló ser pueclo
tomarimirse, Acabóse de
imprimirse, Veinte y nue
ve dias del mes de
Mayo, año de n
cimient de nio
Señor Jesu

Christo,
8,1556
anos

*
End of the work

To the honor and glory of our Lord Jesus Christ and the blessed and glorious Virgin Holy Mary, His mother and our Lady. This is the end of this treatise entitled Sumario còpendioso of the computations of gold and silver necessary in the Kingdoms of Peru. This was published in the magnificent, famous, and most loyal City of Mexico, in the house of Juan Pablos Bressano; with the permission of the illustrious señor Don Luys de Velasco, Viceroy and Governor of New Spain and, also, with the permission of the most illustrious and reverend señor, Brother Alonso de Montufar, archbishop of Mexico, by whom this has been seen and examined and found worthy to be printed. The printing was finished on the twenty-ninth day of the month of May in the year of our Lord Jesus Christ 1556.
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